Değişkenlerine Ayrılabilir Tip Dif. Denklemler :

Örnek 1

$(3x+8)(y^2+4)dx - 4y(x^2+5x+6)dy = 0$ denklemini çözünüz.

$$\frac{(3x+8)(y^2+4)dx}{(y^2+4)} = \frac{4y(x^2+5x+6)dy}{(y^2+4)}$$

$$\frac{(3x+8)dx}{(x^2+5x+6)} = \frac{4y(x^2+5x+6)}{(y^2+4)(x^2+5x+6)}dy$$

$$\int \frac{3x+8}{x^2+5x+6}dx = \int \frac{4y}{y^2+4}dy$$

$$\int \frac{3x+8}{x^2+5x+6}dx = \int \frac{3x+8}{(x+2)(x+3)}dx \; ; \quad \frac{3x+8}{(x+2)(x+3)} = \frac{A}{\underset{(x+3)}{x+2}} + \frac{B}{\underset{(x+2)}{x+3}} = \frac{Ax+3A+Bx+2B}{(x+2)(x+3)}$$

$$\left\{ \begin{array}{l} \bullet\ (A+B)x = 3x \\ \qquad A+B=3 \\ \bullet\ 3A+2B=8 \end{array} \right.$$

$2\underbrace{(A+B)}_{3}+A=8$

$6+A=8$

$A=2$ ve $B=1$ bulunur.

$$\int \frac{3x+8}{(x+2)(x+3)}dx = \int \frac{2}{x+2}dx + \int \frac{1}{x+3}dx \Rightarrow 2\ln(x+2) + \ln(x+3) \text{ olur.}$$

$$\int \frac{4y}{y^2+4}dy \Rightarrow \begin{array}{l} y^2+4=t \\ 2y\,dy=dt \end{array} \quad 2\int \frac{dt}{t} = 2\ln(t) = 2\ln(y^2+4)$$

Düzenlersek,

* c sabit old. için istenilen şekilde yazılabilir.

$$e^{(2\ln(x+2)+\ln(x+3))} = e^{(2\ln(y^2+4)+\ln c)}$$

$$\boxed{(x+2)^2(x+3) = c(y^2+4)^2} \text{ olur.}$$

Örnek 2): $\cos^2x.\cos y\,dx - dy = 0$ diferansiyel denklemini çözünüz.

$$\frac{\cos^2x\cos y\,dx}{\cos y} = \frac{dy}{\cos y} \rightarrow \int \cos^2x\,dx = \int \frac{dy}{\cos y}$$

$\cos 2x = 2\cos^2x - 1$ olduğundan $\cos^2x = \frac{\cos 2x + 1}{2}$ yazılabilir.

$$\int \frac{\cos 2x + 1}{2}dx = \int \frac{\cos 2x}{2}dx + \int \frac{1}{2}dx = \frac{1}{2}\cdot\frac{1}{2}\sin 2x + \frac{x}{2} = \frac{1}{2}\left(\frac{\sin 2x}{2} + x\right)$$

$$\int \frac{dy}{\cos y} = \int \sec y\,dy = \ln(\sec y + \tan y) + c$$

Not:

$$\int \sec ax\,dx = \frac{1}{a}\ln(\sec ax + \tan ax) + c$$

$$\int \operatorname{cosec} ax\,dx = \frac{1}{a}\ln(\operatorname{cosec} ax - \cot x) + c$$

Düzenlersek;

$$\boxed{\frac{1}{2}\left(\frac{\sin 2x}{2} + x\right) = \ln(\sec y + \tan y) + c}$$ olur.

Örnek: $\cos y\,dx + x\sin y\,dy = 0$ denklemini çözünüz.

$$\cos y\,dx = -x\sin y\,dy$$

$$\frac{dy}{dx} = -\frac{\cos y}{x.\sin y} \longrightarrow \int \frac{\sin y\,dy}{-\cos y} = \int \frac{dx}{x}$$

$-\cos y = u$

$\sin y\,dy = du$

$$\Downarrow$$

$$\int \frac{du}{u} = \int \frac{dx}{x}$$

$$\int \frac{du}{u} = \ln u \longrightarrow \ln u = \ln x + \ln c$$

$$e^{\left(\ln(-\cos y)\right)} = e^{\left(\ln x + \ln c\right)}$$

$$\boxed{-\cos y = x + c}$$ bulunur.

Değişkenlerine Ayrılabilir Hale Dönüşebilen Tip Dif. Denklemler

Örnek: **Sınav sorusu**

$y' = e^{x-y-1}$ diferansiyel denklemini çözünüz.

$x-y-1 = u$ dönüşümü yapılırsa;

$1-y' = u'$ olur.

$$1-u' = e^{u} \rightarrow u' = 1-e^{u}$$

$$\frac{du}{dx} = 1-e^{u} \Rightarrow \int \frac{du}{1-e^{u}} = \int dx$$

$$\int \frac{du}{(1-e^{u})} = \int \frac{e^{u}du}{1-e^{u}} + \underbrace{\int \frac{\cancel{1-e^{u}}}{\cancel{1-e^{u}}}du}_{u} \Rightarrow \left.\begin{array}{l} 1-e^{u}=t \\ -e^{u}du=dt \end{array}\right\} \int \frac{-dt}{t} = -\ln(t)$$

Düzenlenirse;

$$-\ln(t) + u = x + c$$

$$\left.\begin{array}{l} t = 1-e^{u} \\ u = x-y-1 \end{array}\right\} t = 1-e^{x-y-1} \left.\right\} -\ln(1-e^{x-y-1}) + x-y-1 = x + c$$

$$\boxed{-\ln(1-e^{x-y-1}) - y - 1 = c}$$ bulunur.

Örnek: $y\,dx-(x-\sqrt{xy})\,dy=0$ denklemini çözünüz.

$$\frac{y\,dx}{x}=\frac{(x-\sqrt{xy})\,dy}{x}$$

$$\frac{y}{x}\,dx=\left(1-\frac{\sqrt{xy}}{x}\right)dy \Rightarrow \frac{dy}{dx}=\frac{y/x}{1-\frac{\sqrt{xy}}{x}}$$

$$\frac{y}{x}=t \rightarrow y'=t'x+t$$

$$t'x+t=\frac{t}{1-\frac{\sqrt{x^2t}}{x}}=\frac{t}{1-\sqrt{t}}$$

$$t'x+t=\frac{t}{1-\sqrt{t}} \Rightarrow t'x=\frac{\cancel{t}-\cancel{t}+t\sqrt{t}}{1-\sqrt{t}}$$

$$t'x=\frac{t\sqrt{t}}{1-\sqrt{t}} \rightarrow \frac{dt}{dx}x=\frac{t\sqrt{t}}{1-\sqrt{t}}$$

Düzenlenirse;

$$\int\frac{1-\sqrt{t}}{t\sqrt{t}}\,dt=\int\frac{dx}{x} \Rightarrow \int\frac{dt}{t\sqrt{t}}-\int\frac{dt}{t}=\int\frac{dx}{x} \rightarrow \int t^{-\frac{3}{2}}dt-\int\frac{dt}{t}=\int\frac{dx}{x}$$

$$\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}-\ln t=\ln x+c$$

$t\rightarrow\frac{y}{x}$ yazılırsa $\quad -2\left(\frac{y}{x}\right)^{-\frac{1}{2}}-\ln\left(\frac{y}{x}\right)=\ln x+c$

$$-2\sqrt{\frac{x}{y}}-\ln\left(\frac{y}{x}\right)=\ln x+c$$

Örnek: $x^2dy-(y^2+2xy)dx=0$ denklemini çözünüz.

$$x^2dy=(y^2+2xy)dx$$

$$x^2\frac{dy}{dx}=y^2+2xy \rightarrow \frac{dy}{dx}=\frac{y^2}{x^2}+\frac{2xy}{x^2}=\left(\frac{y}{x}\right)^2+2\left(\frac{y}{x}\right)$$

$\frac{y}{x}=u$ dönüşümü yapılırsa;

$$y=ux$$
$$y'=u'x+u$$

$$y'=\left(\frac{y}{x}\right)^2+2\left(\frac{y}{x}\right) \Rightarrow u'x+u=u^2+2u$$
$$u'x=u^2+u=u(u+1)$$

$$\frac{du}{dx}x=u(u+1) \longrightarrow \int\frac{du}{u(u+1)}=\int\frac{dx}{x} \rightarrow \int\frac{dx}{x}=\ln x+c$$

$$\int\frac{du}{u(u+1)}=\int\frac{A}{u}du+\int\frac{B}{u+1}du \qquad \underset{(u+1)}{\frac{A}{u}}+\underset{u}{\frac{B}{u+1}}=\frac{1}{u(u+1)} \rightarrow \frac{Au+A+Bu}{u(u+1)}=\frac{1}{u(u+1)}$$

$$(A+B)u=0 \quad A=-B$$
$$A=1 \quad B=-1 \text{ bulunur.}$$

$$\int\frac{du}{u}-\int\frac{du}{u+1}=\ln(u)-\ln(u+1)$$

$u\rightarrow\frac{y}{x}$ yazarsak $\Rightarrow=\ln\left(\frac{y}{x}\right)-\ln\left(\frac{y}{x}+1\right)$

Denklemin çözümü düzenlendiğinde;

$$\boxed{\ln\left(\frac{y}{x}\right)-\ln\left(\frac{y}{x}+1\right)=\ln x+c}$$ bulunur.

Örnek: $(2x+3y+1)dx + (4x+6y+1)dy = 0$ denklemini çözünüz.

$$\frac{dy}{dx} = -\frac{2x+3y+1}{4x+6y+1}$$

$$\left.\begin{aligned} a_1 &= 2 \\ a_2 &= 4 \\ b_1 &= 3 \\ b_2 &= 6 \end{aligned}\right\} \frac{a_1}{a_2} \overset{?}{=} \frac{b_1}{b_2} \rightarrow \frac{2}{4} = \frac{3}{6} = \frac{1}{2} \text{ old.}$$

$2x+3y+1=u$ dönüşümü yapılır.

$$2x+3y+1=u$$

$$2+3y'=u'$$

$$y' = \frac{u'-2}{3} \text{ olur.}$$

Denklemde yerine yazarsak;

$$\frac{u'-2}{3} = -\frac{u}{2u-1}$$

$$u'-2 = -\frac{3u}{2u-1}, \quad u' = \frac{-3u}{2u-1} + 2$$

$$\frac{du}{dx} = \frac{-3u+4u-2}{2u-1} = \frac{u-2}{2u-1} \Rightarrow$$

Düzenlendiğinde;

$$\frac{du}{dx} = \frac{u-2}{2u-1} \rightarrow \int \frac{(2u-1)}{u-2} du = \int dx$$

$$\begin{array}{r|l} 2u-1 & u-2 \\ \mp 2u \pm 4 & 2 \\ \hline 3 & \end{array}$$

$$\frac{2u-1}{u-2} = 2 + \frac{3}{u-2} \text{ yazılabilir.}$$

$$\int 2du + \int \frac{3}{u-2} du = \int dx \rightarrow 2u + 3\ln(u-2) = \ln x + c$$

$$\boxed{2(2x+3y+1) + 3\ln(2x+3y-1) = \ln x + c}$$

olur.

$* \quad xy' = y - 4xe^{\frac{2y}{x}}$?

DEĞİŞKENLERİNE AYRILABİLİR HALE DÖNÜŞTÜRÜLEBİLEN DİF. DENK.

$$y' = \frac{y}{x} - 4e^{\frac{2y}{x}}$$

$$y' - \frac{1}{x}y = -4e^{\frac{2y}{x}}$$

$y = ux$ dönüşümü yapılırsa;

$y' = u'x + u$

$$u'x + u - \frac{1}{x}ux = -4e^{\frac{2ux}{x}} \Rightarrow u'x + \cancel{u} - \cancel{u} = -4e^{2u}$$

$$u'x = -4e^{2u}$$

$$\int \frac{dx}{x} = \int \frac{du}{-4e^{2u}} \longleftarrow \frac{du}{dx} \cdot x = -4e^{2u}$$

$$\ln|x| + c = -\frac{1}{4}\int e^{-2u}du = -\frac{1}{4} \cdot \frac{e^{-2u}}{-2} = \frac{1}{8}e^{-2u} = \frac{1}{8e^{2u}}$$

$u = \frac{y}{x}$ geri dönüşümü yapılırsa;

$\ln|x| + c = \frac{1}{8e^{\frac{2y}{x}}}$ elde edilir.

$$\boxed{\ln|x| + c = \frac{1}{8e^{\frac{2y}{x}}}}$$

$$* \sqrt{1+x^2}\,y' - 3x\sqrt{y^2-1} = 0$$

DEGİŞKENLERİNE AYRILABİLİR DİF. DENKLEM

$$\sqrt{1+x^2}\,\underbrace{y'}_{\frac{dy}{dx}} = 3x\sqrt{y^2-1}$$

$$\frac{dy}{dx} = \frac{3x\sqrt{y^2-1}}{\sqrt{1+x^2}} \Rightarrow \frac{3x}{\sqrt{1+x^2}}dx = \frac{dy}{\sqrt{y^2-1}}$$

$$\int \frac{3x}{\sqrt{1+x^2}}dx = \int \frac{dy}{\sqrt{y^2-1}}$$

Not: $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x + \sqrt{y^2-a^2}\right)$

$1+x^2 = u$

$2x\,dx = du$

$$3\int \frac{du}{2\sqrt{u}} = \ln\left(y+\sqrt{y^2-1}\right) + c$$

$$3\sqrt{u} = \ln\left(y+\sqrt{y^2-1}\right) + c$$

$$\boxed{3\sqrt{1+x^2} = \ln\left(y+\sqrt{y^2-1}\right) + c}$$

DEĞİŞKENLERİNE AYRILABİLİR TİP DİF. DENKLEM

* $y' = x.\sin x.\cos y$

$$\frac{dy}{dx} = x\sin x\cos y \implies x\sin x\,dx = \frac{dy}{\cos y}$$

$$\int x\sin x\,dx = \int \frac{dy}{\cos y} \Rightarrow \int \frac{dy}{\cos y} = \int \sec y\,dy = \operatorname{arctanh}\left(\tan\frac{y}{2}\right)$$

$\Downarrow$

$x = u$; $\sin x\,dx = dv$

$dx = du$, $-\cos x = v$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + c$$

Not:

$$\int \sec(\beta x)\,dx = \frac{\operatorname{arctanh}\left(\tan\frac{\beta x}{2}\right)}{\beta} +$$

$$\boxed{-x\cos x + \sin x + c = \operatorname{arctanh}\left(\tan\frac{y}{2}\right)}$$

$* \quad \dfrac{dy}{dx} = \dfrac{3x-4y-5}{x+2y} \quad ?$

HOMOJEN HALE DÖNÜŞ. TİP DİF. DENK.

$\dfrac{a_1}{a_2} \overset{?}{=} \dfrac{b_1}{b_2} \Rightarrow \dfrac{3}{1} \neq -\dfrac{4}{2} \Rightarrow$ homojen hale dönüştürülebilir.

$$\left.\begin{aligned} X &= x + x_0 \\ Y &= y + y_0 \end{aligned}\right\}$$

x_0 ve y_0'ı bulmak için;

$$\frac{dY}{dX} = \frac{3x_0 - 4y_0 - 5}{x_0 + 2y_0} \Rightarrow \begin{aligned} 3x_0 - 4y_0 - 5 &= 0 \\ 2 \times / \; x_0 + 2y_0 &= 0 \end{aligned}$$

$$\left\{\begin{aligned} + \quad 2x_0 + 4y_0 &= 0 \\ \hline 5x_0 = 5 &\rightarrow \boxed{x_0 = 1} \\ 1 + 2y_0 &= 0 \\ 2y_0 &= -1 \\ \boxed{y_0 = -\frac{1}{2}} \end{aligned}\right.$$

$$\frac{dY}{dX} = \frac{3X - 4Y}{X + 2Y} \Longleftarrow \begin{aligned} X &= x + 1 \\ Y &= y - \frac{1}{2} \end{aligned}$$

(homojen tip dif. denk.)

$Y = uX$ dönüşümü yapılırsa;

$Y' = u'X + u$

$$u'X + u = \frac{3X - 4uX}{X + 2uX} = \frac{\cancel{X}(3-4u)}{\cancel{X}(1+2u)} \Rightarrow u'X = \frac{3-4u}{1+2u} - u$$

$$u'X = \frac{3 - 4u - u - 2u^2}{1+2u}$$

$$\frac{du}{dX} X = \frac{-2u^2 - 5u + 3}{2u+1}$$

$$\int \frac{(2u+1)\,du}{-2u^2 - 5u + 3} = \int \frac{dX}{X}$$

$$\frac{2u+1}{(1-2u).(u+3)} = \frac{A}{\underset{(u+3)}{1-2u}} + \frac{B}{\underset{(1-2u)}{u+3}} \Rightarrow Au+3A+B-2Bu = 2u+1$$

$$(A-2B)\cancel{u} = 2\cancel{u}$$

$$A-2B=2 \quad \ldots (1)$$

$$3A+B=1 \quad \ldots (2)$$

$$A = 2+2B$$

$$3(2+2B)+B=1 \Rightarrow 6+7B=1$$

$$7B=-5$$

$$\left.\boxed{B=-\frac{5}{7}}\right\} \quad \boxed{A = 2-\frac{10}{7} = \frac{4}{7}}$$

$$-\frac{5}{7}\int \frac{1}{1-2u}\,du + \frac{4}{7}\int \frac{du}{u+3} = \int \frac{dx}{x}$$

$$1-2u=t$$

$$-2\,du = dt$$

$$\frac{5}{7}\int \frac{dt}{2t} \Rightarrow \frac{5}{14}\ln|t| = \frac{5}{14}\ln|1-2u| \quad \text{olur.}$$

İntegraller alındığında;

$$\frac{5}{14}\ln|1-2u| + \frac{4}{7}\ln|u+3| = \ln|x| + c$$

$u = \frac{y}{x}$ yazarsak

$$\boxed{\left(1-\frac{2y}{x}\right)^{5/14} + \left(\frac{y}{x}+3\right)^{4/7} = x + c} \quad \text{bulunur.}$$

HOMOJEN TİP
DİF. DENK.

$$* \quad (1+2e^{x/y})dx + 2e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$$

$z = \dfrac{x}{y}$ dönüşümü yaparsak; (kolaylık sağlaması açısından)

$x = zy$

$dx = ydz + zdy$ olur.

Denklemde yaptığımız dönüşümü yerine yazarsak;

$$(1+2e^z)(ydz+zdy) + 2e^z(1-z)dy = 0$$

$$ydz + zdy + 2e^z ydz + \cancel{2e^z z dy} + 2e^z dy - \cancel{2ze^z dy} = 0$$

$$(y+2e^z y)dz + (z+2e^z)dy = 0$$

$$\frac{dy}{dz} = -\frac{y(1+2e^z)}{z+2e^z} \Rightarrow \left(\frac{1+2e^z}{z+2e^z}\right)dz = -\frac{dy}{y}$$

$$\int \frac{1+2e^z}{z+2e^z}dz = \int -\frac{dy}{y}$$

$z + 2e^z = t$

$(1+2ze^z)dz = dt$ olur.

$$\int \frac{dt}{t} = \int -\frac{dy}{y}$$

$$\ln|t| = -\ln|y| + \ln c$$

$$\ln|z+2e^z| = -\ln|y| + \ln c$$

$z = \frac{x}{y}$ geri dönüşümü yapılırsa;

$$\ln\left|\frac{x}{y} + 2e^{x/y}\right| = -\ln|y| + \ln c$$

$$\ln\left|\left(x/y + 2e^{x/y}\right) \cdot y\right| = \ln c$$

$$\boxed{x + 2ye^{x/y} = c}$$ bulunur.

- $\sin x \sin y\,dx + \cos x \cos y\,dy = 0$ dif. denk. genel çözümünü bulunuz.

DEĞİŞKENLERİNE AYRILABİLİR TİP DİF. DENK

$$\left.\begin{array}{ll} \sin y = u & \cos x = t \\ \cos y\,dy = du & -\sin x\,dx = dt \end{array}\right\} -u\,dt + t\,du = 0$$

$$t\,du = u\,dt \Rightarrow \int \frac{du}{u} = \int \frac{dt}{t}$$

$$\ln u = \ln t + \ln c$$

$$e^{(\ln \sin y)} = e^{(\ln \cos x + \ln c)} \Rightarrow \boxed{\sin y = \cos x + c}$$ olur.

- $(y+1)\dfrac{dy}{dx} = x^2 - x^2(y^2+2y+5)$ dif. denk. genel çözümünü bulunuz.

$$\frac{dy}{dx} = \frac{x^2(1-y^2-2y-5)}{y+1} = \frac{-x^2(y^2+2y+4)}{y+1}$$

$$\int \frac{(y+1)\,dy}{(y^2+2y+4)} = \int -x^2\,dx$$

$$\left.\begin{array}{l} y^2+2y+4 = u \\ (2y+2)\,dy = du \\ 2(y+1)\,dy = du \\ (y+1)\,dy = \dfrac{du}{2} \end{array}\right\} \int \frac{du}{2u} = \int -x^2\,dx$$

$$\frac{1}{2}\ln u = -\frac{x^3}{3} + c$$

$$\boxed{\frac{1}{2}\ln(y^2+2y+4) + \frac{x^3}{3} = c}$$

- $y\,dx - x\,dy = x(dy - y\,dx)$ dif. denk. çözünüz.

Değişkenlerine Ayrılabilen Tip

$y\,dx - x\,dy = x\,dy - xy\,dx$

$(y + xy)\,dx = (x + x)\,dy$

$y(1+x)\,dx = 2x\,dy \longrightarrow \dfrac{dy}{dx} = \dfrac{y(1+x)}{2x} \longrightarrow \displaystyle\int \frac{dy}{y} = \int \frac{1+x}{2x}\,dx$

$$\int \frac{dy}{y} = \int \frac{1}{2x}\,dx + \int \frac{1}{2}\,dx$$

$$\boxed{\ln y = \frac{1}{2}\ln x + \frac{1}{2}x + c}$$

- $y' = \dfrac{3x^2 + 2x + 1}{2(y+1)}$ dif. denk. çözünüz.

$\dfrac{dy}{dx} = \dfrac{3x^2 + 2x + 1}{2(y+1)} \qquad \displaystyle\int 2(y+1)\,dy = \int (3x^2 + 2x + 1)\,dx$

$$\int 2y\,dy + \int 2\,dy = \int 3x^2\,dx + \int 2x\,dx + \int dx$$

$$\frac{2y^2}{2} + 2y = \frac{3x^3}{3} + \frac{2x^2}{2} + x + c$$

$$\boxed{y^2 + 2y = x^3 + x^2 + x + c}$$

- $y \ln x \ln y \, dx + dy = 0$ dif. denkleminin genel çözümünü bulunuz.

DEĞİŞKENLERİNE AYRILABİLİR TİP DİF. DENK.

$$\frac{y \ln x \ln y \, \cancel{dx}}{\cancel{dx}} = -\frac{dy}{dx}$$

$$y \ln x \ln y = -\frac{dy}{dx} \Rightarrow \int \ln x \, dx = \int -\frac{dy}{y \ln y}$$

Kısmi integrasyon ile;

$\ln x = u \qquad dx = dv$

$\frac{1}{x} dx = du \qquad x = v$

$$\int u \, dv = uv - \int v \, du$$

$$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} dx = x \ln x - x \text{ dır.}$$

$\ln y = t$

$\frac{1}{y} dy = dt$

$$= \int -\frac{dt}{t} = -\ln|t| = -\ln(\ln y)$$

$$x \ln x - x = -\ln(\ln y) + c$$

$$\boxed{x(\ln x - 1) + \ln(\ln y) = c \quad \text{dur.}}$$

- $\left(x+y\cot\frac{x}{y}\right)dy - y\,dx = 0$ dif. denk. çözünüz. Homojen tip.

$\frac{\left(x+y\cdot\cot\frac{x}{y}\right)dy = y\,dx}{y \qquad\qquad y} \rightarrow \left(\frac{x}{y}+\cot\frac{x}{y}\right)dy = dx$

$\frac{x}{y}=u$ dersek; $\qquad \frac{dy}{dx}=\frac{1}{\frac{x}{y}+\cot\frac{x}{y}}$

$y=\frac{x}{u}$

$y'=\frac{u-u'x}{u^2}$ olur.

Denklemde yerine yazarsak;

$\frac{u-u'x}{u^2}=\frac{1}{u+\cot u} \rightarrow u-u'x=\frac{u^2}{u+\cot u}$

$u'x = u-\frac{u^2}{u+\cot u}=\frac{u^2+u\cot u-u^2}{u+\cot u}=\frac{u\cot u}{u+\cot u}$

$\frac{du}{dx}x=\frac{u\cot u}{u+\cot u} \rightarrow \int\frac{u+\cot u}{u\cot u}du=\int\frac{dx}{x}$

$\int\frac{u}{u\cot u}du+\int\frac{\cot u}{u\cot u}du=\int\frac{dx}{x}$

$\int\frac{\sin u\,du}{\cos u}+\underbrace{\int\frac{du}{u}}_{\ln u}=\underbrace{\int\frac{dx}{x}}_{\ln x}$

$\cos u = z$

$-\sin u\,du = dz \rightarrow \int -\frac{dz}{z}=-\ln z=-\ln(\cos u)$

$u\rightarrow\frac{x}{y}$ yazarsak;

$-\ln\left(\cos\frac{x}{y}\right)+\ln\frac{x}{y}=\ln x+\ln c$

$\left(\cos\frac{x}{y}\right)^{-1}\cdot\left(\frac{x}{y}\right)=x\cdot c$

$\frac{x/y}{\cos\left(\frac{x}{y}\right)}=x.c$ olur.

— Sınav Sonrası —

$(y^2+2y)dy + (y^3+3y^2-3)x\,dx = 0$ dif. denk. genel çözümünü bulunuz.

Denklemde dy/dx ifadesini elde edelim ve denklemi düzenleyelim.

$$(y^2+2y)\frac{dy}{dx} + (y^3+3y^2)x = 3x$$

$y^3+3y^2 = u$

$(3y^2+6y)y' = u'$ olur.

$\Big\}$ $3(y^2+2y)y' = u'$ olacağından;

7

$$\frac{u'}{3} + ux = 3x \longrightarrow \boxed{u' + 3xu = 9x}$$ [Lineer tip dif. denk.]

$f(x) = 3x$

$g(x) = 9x$ olacaktır.

3

$$u = \left[\int g(x)\, e^{\int f(x)\,dx}\,dx + c\right] e^{-\int f(x)\,dx}$$

$$u = \left[\int 9x\, e^{\int 3x\,dx}\,dx + c\right] e^{\int -3x\,dx}$$ 3 ; $\int 3x\,dx = \frac{3x^2}{2}$

$$= \left[\int 9x\, e^{\frac{3}{2}x^2}\,dx + c\right] e^{-\frac{3}{2}x^2} \Longrightarrow \quad \frac{3}{2}x^2 = t$$

$$\frac{3}{2}\cdot 2x\,dx = dt$$

$$= \left[\int 3e^t\,dt + c\right] e^{-\frac{3}{2}x^2} \Rightarrow$$ Buradan;

$$u = \left[3e^t + c\right] e^{-\frac{3}{2}x^2}$$ 2

$$u = \left[3e^{\frac{3}{2}x^2} + c\right] e^{-\frac{3}{2}x^2} = 3 + ce^{-\frac{3}{2}x^2}$$ olur.

$u = y^3 + 3y^2$ geri dönüşümü yapılırsa;

$$\boxed{y^3 + 3y^2 = 3 + ce^{-\frac{3}{2}x^2}}$$ elde edilir.

ya da

$$y^2 = \frac{3 + ce^{-\frac{3}{2}x^2}}{3+y}$$ ~~2~~

- Sınav sorusu -

$$dy + (y - \sin x)\cos x \, dx = 0 \quad ; \quad \mu = \mu(x)$$

Yukarıdaki diferansiyel denklemde;

$$\left.\begin{aligned} P(x,y) &= (y - \sin x)\cos x \\ Q(x,y) &= 1 \end{aligned}\right\} \; P'_y \stackrel{?}{=} Q'_x$$

3

$$\left.\begin{aligned} P'_y &= y\cos x \\ Q'_x &= 0 \end{aligned}\right\} \; P'_y \neq Q'_x$$ olduğundan tam dif. denk. değildir.

İntegrasyon çarpanı $\mu = \mu(x)$ şeklinde verilmiş. O zaman;

$$\frac{dv}{dx} = 1 \quad ve \quad \frac{dv}{dy} = 0 \qquad \left[\begin{aligned} \mu &= f(v) \\ v &= f(x,y) \end{aligned}\right]$$

$$\mu = e^{\int \left[\frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{\frac{\partial v}{\partial y} P - \frac{\partial v}{\partial x} Q}\right]}$$

$$\mu = e^{\int \frac{0 - \cos x}{0 \ldots - 1.1} \partial v} \rightarrow \mu = e^{\int \cos x \, dv} \Rightarrow \mu = e^{\sin x}$$ bulunur. 5

Denklem μ ile çarpılırsa;

$$e^{\sin x} dy + (y - \sin x)\cos x \, e^{\sin x} dx = 0$$

Yeni denklemde;

$$\left.\begin{aligned} P(x,y) &= (y - \sin x)\cos x . e^{\sin x} \\ Q(x,y) &= e^{\sin x} \end{aligned}\right\} \; P'_y \stackrel{?}{=} Q'_x$$

4

$$\left.\begin{aligned} P'_y &= e^{\sin x} . \cos x \\ Q'_x &= e^{\sin x} . \cos x \end{aligned}\right\} \; P'_y = Q'_x$$ olduğundan tam dif. denklemdir.

Tam diferansiyel denklem

Örnek:

$(4x^3+3y^2+\cos x)dx+(6xy+2)dy=0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\left.\begin{aligned} P(x,y) &= 4x^3+3y^2+\cos x \\ Q(x,y) &= 6xy+2 \end{aligned}\right\}$$

$P'_y \stackrel{?}{=} Q'_x$

$6y=6y$ olduğundan tam dif. denk.

$$\int_a^x P(x,y)\,dx+\int Q(a,y)\,dy=c$$

$$\int_a^x (4x^3+3y^2+\cos x)\,dx+\int (6ay+2)\,dy=c$$

$$\left(4\frac{x^4}{4}+3xy^2+\sin x\right)\Big|_a^x+\frac{6ay^2}{2}+2y=c$$

$$x^4+3xy^2+\sin x-a^4-\cancel{3ay^2}-\sin a+\cancel{3ay^2}+2y=c$$

$$\boxed{x^4+3xy^2+\sin x+2y=c}$$

$$(y+1)\frac{dy}{dx}+x^2(y^2+2y+5)=x^2$$ dif. denk. genel çözümünü bulunuz.

$$\underbrace{(y+1)dy}_{dt/2}\frac{1}{dx}+x^2\underbrace{(y^2+2y+5)}_{t}=x^2$$

Bu denklemde; $y^2+2y+5=t$ dönüşümü yapılırsa, $2(y+1)dy=dt$

Dönüşüme göre denklemin yeni hali;

$$\frac{dt}{2dx}+x^2t=x^2$$

$$\frac{dt}{dx}+2x^2t=2x^2 \qquad \left[t'+f(x)t=g(x)\right]$$ Lineer tip dif. denk.

Lineer dif. denklemin genel çözümü;

$$t=\left[\int g(x)e^{\int f(x)dx}dx+c\right]e^{-\int f(x)dx} \quad ; \quad f(x)=2x^2, \quad g(x)=2x^2$$

$$t=\left[\int 2x^2e^{\int 2x^2dx}dx+c\right]e^{-\int 2x^2dx} \quad ; \quad \int 2x^2dx=\frac{2}{3}x^3$$

$$t=\left[\int 2x^2e^{\frac{2}{3}x^3}dx+c\right]e^{-\frac{2}{3}x^3}$$

$$\int 2x^2e^{\frac{2x^3}{3}}dx \ ; ? \quad \Rightarrow \quad \frac{2}{3}x^3=u, \quad \frac{2}{\cancel{3}}\cdot\cancel{3}x^2dx=du$$

$$\int e^u du=e^u+c$$ 'dir.

Geri dönüşümler yapılırsa; $t \Rightarrow y^2+2y+5$

$u \Rightarrow \frac{2}{3}x^3$

$$y^2+2y+5=\left(e^{\frac{2}{3}x^3}+c\right)e^{-\frac{2}{3}x^3}=1+ce^{-\frac{2}{3}x^3}$$

$$\boxed{y^2+2y+4=ce^{-\frac{2}{3}x^3}}$$ olur.

Örnek: $y(6y^2-x-1)dx+2x\,dy=0$ dif. denk. genel çözümünü bulunuz.

Bernoulli dif. denklemi

$$\frac{y(6y^2-x-1)}{2x}+\frac{2x\frac{dy}{dx}}{2x}=0$$

$$\frac{dy}{dx}+\frac{y(6y^2-x-1)}{2x}=0 \longrightarrow \frac{dy}{dx}+\frac{3}{x}y^3-\frac{y}{2}-\frac{y}{2x}=0$$

$$\frac{dy}{dx}+\left(-\frac{1}{2}-\frac{1}{2x}\right)y=-\frac{3}{x}y^3 \longrightarrow \text{Bernoulli diferansiyel denklemi}$$

$n=3 \rightarrow y^{1-3}=y^{-2}=t$ dönüşümü yapılırsa

$$t'-2\left(-\frac{1}{2}-\frac{1}{2x}\right)t=\frac{6}{x} \longrightarrow \text{Lineer dif. denk}$$

$$t'+\left(1+\frac{1}{x}\right)t=\frac{6}{x}$$

$$t=\frac{1}{y^2}=\left[\int\frac{6}{x}e^{\int(1+\frac{1}{x})dx}dx+c\right]e^{-\int(1+\frac{1}{x})dx} \qquad \int\left(1+\frac{1}{x}\right)dx=x+\ln x$$

$$=\left[\int\frac{6}{\cancel{x}}e^x\cdot\cancel{e^{\ln x}}dx+c\right]\frac{1}{e^x.x} \longrightarrow \left[\int 6e^x dx+c\right]\frac{1}{xe^x}$$

$$\frac{1}{y^2}=\frac{6e^x}{xe^x}+\frac{c}{e^x.x}=\frac{6}{x}+\frac{c}{e^x.x}$$

$$\boxed{y^2=\frac{1}{\frac{6}{x}+\frac{c}{e^x.x}}}$$

$y^2 e^{2x} dx - (e^{2x}+4) dy = 0$ Sınav sorusu diferansiyel denk. genel çözümünü bulunuz.

$$y^2 e^{2x} dx = (e^{2x}+4) dy$$

$$\int \frac{e^{2x}\, dx}{(e^{2x}+4)} = \int \frac{dy}{y^2} \quad 5$$

$$\left.\begin{array}{l} e^{2x} = u \quad 4 \\ 2e^{2x} dx = du \end{array}\right\} \quad \int \frac{du}{2(u+4)} = \int \frac{dy}{y^2} \quad 2$$

$$\left.\begin{array}{l} \frac{1}{2}\int \frac{du}{u+4} = \frac{1}{2}\ln(u+4) \text{ olur.} \quad 4 \\ \int \frac{dy}{y^2} = -\frac{1}{y} \text{ olur.} \end{array}\right\} \quad \frac{1}{2}\ln(e^{2x}+4) + \frac{1}{y} = C \quad \cancel{2}$$

elde edilir.

Örnek: (x'in bulunmadığı)

$$y(y-1)y'' + y'^2 = 0$$

$$y' = P \qquad y'' = P\frac{dP}{dy}$$

$$y(y-1)P\frac{dP}{dy} + P^2 = 0$$

$$\int \frac{dP}{P} + \int \frac{dy}{y(y-1)} = 0 \quad \rightarrow \ln P + \int \frac{A}{y}dy + \int \frac{B}{y-1}dy = 0$$

$$\frac{A}{y} + \frac{B}{y-1} = \frac{1}{y(y-1)} \rightarrow Ay - A + By = 1$$

$$\boxed{A = -1}$$

$$A + B = 0$$

$$\boxed{B = 1}$$

$$\ln P - \ln y + \ln(y-1) = \ln c$$

$$\ln \frac{P}{c} = \ln \frac{y}{y-1} \rightarrow P = \frac{cy}{y-1}$$

$$\frac{dy}{dx} = \frac{cy}{y-1} \quad \rightarrow \int \frac{y-1}{y}dy = \int c\,dx$$

$$\boxed{y - \ln y = c_1 x + c_2}$$

Örnek:

$y = xy' + 4y'^2$ dif. denk. çözünüz

$y = xy' + \varnothing(y') \rightarrow$ (clairaut)

$y' = c$ yazarsak (c=sbt)

$y = xc + 4c^2$

$y_c' = 0$ yapılırsa;

$y_c' = x + 8c = 0$

$\boxed{c = -\frac{x}{8}}$

$y = x \cdot -\frac{x}{8} + 4\frac{x^2}{64} \rightarrow \boxed{y^2 = -\frac{x^2}{8} + \frac{x^2}{16}}$

– Sinav sorusu –

Integrasyon Çarpanı Metodu

$x\,dy - (x^2+y^2+y)\,dx = 0$ dif. denkleminin $\mu = \mu(x^2+y^2)$ şeklinde bir integrasyon çarpanı varsa, bu integrasyon çarpanını bularak dif. denklemin genel çözümünü elde ediniz.

$$\left.\begin{aligned} P(x,y) &= -(x^2+y^2+y) \\ Q(x,y) &= x \end{aligned}\right\} \quad \begin{aligned} P'_y &\overset{?}{=} Q'_x \\ -2y-1 &\neq 1 \end{aligned}$$

2

old. tam dif. denk. değildir.

Integrasyon çarpanı bulunarak, denklem tam dif. denk. haline dönüştürülmelidir.

$$\mu = e^{\int \left[\frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{P\frac{\partial v}{\partial y} - Q\frac{\partial v}{\partial x}}\right]\partial v} \quad ; \quad v = x^2+y^2 \quad \frac{\partial v}{\partial x} = 2x \text{ ve } \frac{\partial v}{\partial y} = 2y \text{ olacaktır.}$$

2

$$\mu = e^{\int \frac{2y+2}{-2yx^2-2y^3-2y^2-2x^2}\partial(x^2+y^2)} = e^{\int \frac{2y+2}{-2(x^2+y^2)-2y(x^2+y^2)}\partial(x^2+y^2)}$$

Elde edilen ifade düzenlenirse,

$$e^{\int \frac{\cancel{2y+2}}{-\cancel{(2y+2)}(x^2+y^2)}d(x^2+y^2)} = e^{-\int \frac{\partial(x^2+y^2)}{x^2+y^2}} = e^{-\ln(x^2+y^2)} = (x^2+y^2)^{-1}$$

6

$$\mu = \frac{1}{x^2+y^2} \quad \text{bulunur.}$$

Denklemin her tarafı "$\frac{1}{x^2+y^2}$" ifadesi ile çarpılıp, denklem tam dif. hale getirilmelidir.

$$\frac{x}{x^2+y^2}dy - \left(\frac{x^2+y^2+y}{x^2+y^2}\right)dx = 0 \Rightarrow \frac{x}{x^2+y^2}dy - \left(1+\frac{y}{x^2+y^2}\right)dx = 0 \quad (1)$$

Denklem (1)'de ;

$$\left.\begin{aligned} P(x,y) &= -\left(1+\frac{y}{x^2+y^2}\right) \\ Q(x,y) &= \frac{x}{x^2+y^2} \end{aligned}\right\} \quad \begin{aligned} P_y' &\overset{?}{=} Q_x' \\ \frac{-x^2+y^2}{(x^2+y^2)^2} &= \frac{y^2-x^2}{(x^2+y^2)^2} \text{ old. tam dif. denklemdir.} \end{aligned}$$

2

Genel çözüm ifadesi;

3

$$\int_a^x P(x,y)dx + \int Q(a,y)dy = C \Rightarrow \int_a^x -\left(1+\frac{y}{x^2+y^2}\right)dx + \int \frac{a}{a^2+y^2}dy = C$$

$$\int_a^x -dx + \int_a^x \frac{y}{x^2+y^2}dx + \int \frac{a}{a^2+y^2}dy = C$$

Hatırlatma;

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\arctan\frac{x}{a}$$

Integraller alındığında;

$$\left[-x - y.\frac{1}{y}\arctan\frac{x}{y}\right]\Bigg|_a^x + \cancel{a}.\frac{1}{\cancel{a}}\arctan\frac{y}{a} = C$$

2

$$-x + a - \arctan\frac{x}{y} + \arctan\frac{a}{y} + \arctan\frac{y}{a} = C$$

$$-x - \arctan\frac{x}{y} = C$$

Genel çözüm

$$\boxed{x + \arctan\frac{x}{y} = C} \text{ olur.}$$

$(x\cos y-\sin^2 y)dy-\sin y\,dx=0$ dif. denkleminin $\mu=\mu(y)$ şeklinde bir integrasyon çarpanı varsa, bu integrasyon çarpanını bularak dif. denklemin genel çözümünü elde ediniz.

İntegrasyon Çarpanı Metodu

$$\left.\begin{aligned} P(x,y)&=-\sin y \\ Q(x,y)&=x\cos y-\sin^2 y\end{aligned}\right\} \quad \begin{aligned} P_y' &\overset{?}{=} Q_x' \\ -\cos y &\neq \cos y \end{aligned}$$ olduğundan tam dif. denk. değildir.

İntegrasyon çarpanını bulmak için;

$$\mu=e^{\int\left[\frac{\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}}{P\frac{\partial v}{\partial y}-Q\frac{\partial v}{\partial x}}\right]\partial v} \quad ; \quad \frac{\partial v}{\partial x}=0 \text{ ve } \frac{\partial v}{\partial y}=1$$

$$\mu=e^{\int\left(\frac{\cos y+\cos y}{-\sin y.1-0}\right)dy}=e^{\int\frac{2\cos y\,dy}{-\sin y}} \quad \begin{aligned}\sin y&=t\\ \cos y\,dy&=dt\end{aligned} \Rightarrow e^{-2\int\frac{dt}{t}}=e^{-2\ln t}$$

$t=\sin y$ geri dönüşümü yapılırsa,

$$\left.\mu=e^{-2\ln(t)}=e^{-2\ln(\sin y)}=\frac{1}{\sin^2 y}\right\} \quad \mu=\frac{1}{\sin^2 y}$$ bulunur.

• Denklemin genel çözümü için; her taraf " $\frac{1}{\sin^2 y}$ " ile çarpılmalıdır.

$$\frac{x\cos y-\sin^2 y}{\sin^2 y}dy-\frac{\sin y}{\sin^2 y}dx=0 \Rightarrow \left(\frac{x\cos y}{\sin^2 y}-1\right)dy-\frac{1}{\sin y}dx=0 \quad (A)$$

Yeni denklemde (A denklemi)

$P(x,y)=-\frac{1}{\sin y}$

$Q(x,y)=\frac{x\cos y}{\sin^2 y}-1$

$P_y' \stackrel{?}{=} Q_x'$

$$\left.\begin{aligned} &\frac{-(-\cos y)}{\sin^2 y} = \frac{\cos y}{\sin^2 y} = P_y' \\ &\frac{\cos y}{\sin^2 y} = Q_x' \end{aligned}\right\} \quad P_y' = Q_x' \text{ old. tam dif. denklemdir.}$$

$$\int_a^x P(x,y)\,dx + \int Q(a,y)\,dy = C \quad \ldots\ldots \text{ Tan dif. denklenin genel çözümü}$$

$$\int_a^x -\frac{1}{\sin y}\,dx + \boxed{\int \frac{a\cos y}{\sin^2 y}\,dy} - \int dy = C$$

$$\left.\begin{aligned} \sin y &= u \\ \cos y\,dy &= du \end{aligned}\right\} \int \frac{a.\,du}{u^2} = \frac{a.u^{-1}}{-1} = -\frac{a}{u} = -\frac{a}{\sin y}$$

$$-\frac{x}{\sin y}\Bigg|_a^x + \frac{a}{\sin y} - y = C$$

$$-\frac{x}{\sin y} + \frac{a}{\sin y} + \frac{a}{\sin y} - y = C$$

Genel çözüm;

$$\boxed{y + \frac{(2a+x)}{\sin y} = C}$$

Örnek: $y = -xy' + (y')^2$ dif. denk. çözünüz. (Lagrange)

$y' = P$

$y = x\varphi(y') + \varnothing(y')$

$y = -xP + P^2$

$(y = xy' + \varnothing y' = \text{clairaut})$

$$y'_x = P = -P - x\frac{dP}{dx} + 2P\frac{dP}{dx}$$

$$2P = \frac{dP}{dx}(2P - x)$$

$$2P\frac{dP}{dx} + x - 2P = 0$$

$$\frac{dx}{dP} + \frac{1}{2P}x = 1 \quad (\text{Lineer})$$

$$x = \left[\int 1 e^{\int \frac{1}{2P}dP} dP + c\right] e^{-\int \frac{1}{2P}dP}$$

$$\int \frac{1}{2P}dP = \frac{1}{2}\ln P$$

$$e^{\int \frac{1}{2P}dP} = e^{\frac{1}{2}\ln P} = P^{1/2}$$

$$x = \left[\int P^{1/2}dP + c\right] P^{-1/2}$$

$$x = \left[\frac{P^{\frac{3}{2}}}{\frac{3}{2}} + c\right] P^{1/2}$$

$$\boxed{x = \frac{2}{3}P^2 + cP^{1/2}}$$

Örnek:

(y'nin bulunmadığı)

$y'' - xy' = xe^{x^2/2}$ dif. denk. çözünüz.

$y' = p \ , \ y'' = p'$

$p' - xp = xe^{x^2/2}$ } Lineer dif. denklem

$$p = \left[\int xe^{x^2/2} e^{\int -x\,dx} dx + c\right] e^{\int x\,dx}$$

$$p = \left[\int xe^{x^2/2} e^{-x^2/2} dx + c\right] e^{x^2/2}$$

$$= \left[\int x\,dx + c\right] e^{x^2/2} \rightarrow p = \left(\frac{x^2}{2} + c\right) e^{x^2/2}$$

$$p = \frac{dy}{dx} = \left(\frac{x^2}{2} + c\right) e^{x^2/2}$$

$$\int dy = \int \left(\frac{x^2 e^{x^2/2}}{2} + ce^{x^2/2}\right) dx$$ olur.

BERNOULLI DİF. DENK.

* $xy' + y = y^2 \ln x \qquad y' + f(x) y = g(x) y^n$

$$y' + \frac{y}{x} = \frac{\ln x}{x} y^2 \Rightarrow \text{Bernoulli dif. denk.}$$

$n=2 \rightarrow y^{1-n} = y^{1-2} = y^{-1} = t$ dönüşümü yapılırsa;

$$t' + (1-n) f(x) t = g(x)$$

$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{\ln x}{x}$$

$$t' - \frac{1}{x} t = -\frac{\ln x}{x} \Rightarrow \text{lineer dif. denk.}$$

$$t = \frac{1}{y} = \left[\int -\frac{\ln x}{x} e^{\int -\frac{1}{x} dx} dx + c \right] e^{\int \frac{1}{x} dx}$$

$$t = \frac{1}{y} = \left[\int -\frac{\ln x}{x} \underbrace{e^{-\ln x}}_{x^{-1}} dx + c \right] \underbrace{e^{\ln x}}_{x}$$

$$\frac{1}{y} = \left[\int -\frac{\ln x}{x^2} dx + c \right] x \quad ;$$

$$\int \frac{\ln x \, dx}{x^2} = ?$$

$-\frac{1}{x^2} dx = dv \qquad \ln x = u$

$1/x = v \qquad \frac{1}{x} dx = du$

$$\int u \, dv = uv - \int v \, du$$

$$\int -\frac{\ln x \, dx}{x^2} = \ln x \cdot \frac{1}{x} - \int \frac{1}{x} \cdot \frac{1}{x} dx$$

$$= \frac{\ln|x|}{x} + \frac{1}{x} \quad \text{olur.}$$

$$\frac{1}{y} = \left[\frac{\ln|x| + 1}{x} + c \right] x \Rightarrow \frac{1}{y} = \ln|x| + 1 + cx$$

$$\boxed{y = \frac{1}{\ln|x| + 1 + cx}}$$

$\left[x\cos^2(y/x) - y\right]dx + x\,dy = 0$ dif. denkleminin genel çözümünü bulunuz.

$$\frac{dy}{dx} = -\frac{x\cos^2\left(\frac{y}{x}\right) - y}{x} \quad ; \quad \frac{y}{x} = u \text{ dönüşümü yaparsak,}$$

$$= -\cos^2\left(\frac{y}{x}\right) + \frac{y}{x} \qquad y = ux$$

$$y' = u'x + u$$

$$u'x + \cancel{u} = -\cos^2 u + \cancel{u} \rightarrow u'x = -\cos^2 u$$

$$x\frac{du}{dx} = -\cos^2 u$$

$$\int \frac{du}{\cos^2 u} = \int -\frac{dx}{x}$$

$\frac{1}{\cos^2 u} = \sec^2 u$ olduğundan

$$\int \sec^2 u \, du = -\int \frac{dx}{x}$$

$$\tan u = -\ln x + c$$

$$\boxed{\tan\left(\frac{y}{x}\right) = -\ln x + c}$$