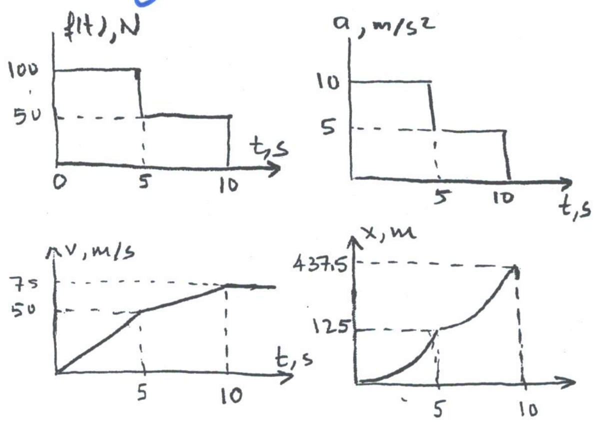
BÖLÜM 2

2.1



$t = 15$ s de hız 75 m/s

$0 - V = 0 \quad -75 = at = a\,5\,s \quad a = -15\ m/s^2$

$F = ma = 10(-15) = -150\ N$

2.2

$M_{toplam} = 10 + 10 = 20\ kg$

$F = 10\ kg \times 10\ m/s^2 = 100\ N$

$$a = \frac{F}{M_{top}} = \frac{100\ N}{20\ kg} = 5\ m/s^2$$

$$V = at = 5\,\frac{m}{s^2}\cdot 5\,s = 25\ m/s$$

$$x = \frac{1}{2}at^2 = \frac{1}{2}5\,\frac{m}{s^2}\cdot 5^2 s^2 = 62.5\ m$$

2.3

$$E_k = pv - E_k^* \qquad p = \frac{m_o v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E_k = \frac{m_o v^2}{\sqrt{1-\frac{v^2}{c^2}}} - \int_0^v \frac{m_o v\,dv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{m_o v^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_o c\int_0^v \frac{v\,dv}{\sqrt{c^2-v^2}}$$

$$= \frac{m_o v^2}{\sqrt{1-\frac{v^2}{c^2}}} - \left[m_o c\left(-\sqrt{c^2-v^2}\right)\right]\Bigg|_{v=0}^{v=v}$$

$$= \frac{m_o v^2}{\sqrt{1-\frac{v^2}{c^2}}} + m_o c\sqrt{c^2-v^2} - m_o c^2$$

$$= \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_o c^2 = m_o c^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

Proton kütlesi $= 1.66\times10^{-24}$ gr

Proton hızı $= 0.99c \qquad c = 3\times10^8\ m/s$

$$E_k = 1.66\times10^{-27}\,kg\,(3\times10^8\ m/s)^2\left[\frac{1}{\sqrt{1-\frac{(0.99)^2c^2}{c^2}}} - 1\right]$$

$$= 1.66\times10^{-27}\times9\times10^{16}(7.0888-1) = 9.0966\times10^{-10}$$

$$= 9.0966\times10^{-10}\ N\cdot m = 9.0966\times10^{-10}\ J$$

$$p = \frac{m_o v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.66\times10^{-27}\,kg\times0.99\times3\times10^8\ m/s}{\sqrt{1-\frac{0.99^2c^2}{c^2}}}$$

$$= 3.495\times10^{-18}\ kg\cdot m/s$$

2.4

a) $J = \frac{1}{2}Mr^2$ b) $J = Mr^2$

c) $J = \frac{1}{2}Mr^2$ d) $J = \frac{1}{4}mr^2$

f) $J = \frac{1}{6}ML^2$

g) $J = 2mb^2 + 2mc^2 + 4ma^2$

2.5

$$M_T(t) = M + m_o - C_m t$$

$$a(t) = \frac{F_o - (M+m_o-C_m t)g}{M+m_o-C_m t}$$

$$= \frac{F_o}{M+m_o-C_m t} - g$$

$$v(t) = \int_0^t\left[\frac{F_o}{M+m_o-C_m\tau} - g\right]d\tau$$

$$= \left[-\frac{F_o}{C_m}\ln(M+m_o-C_m\tau) - g\tau\right]_{\tau=0}^{\tau=t}$$

$$= -\frac{F_o}{C_m}\left[\ln(M+m_o-C_m t) - \ln(M+m_o)\right] - gt$$

$$= -\frac{F_o}{C_m}\ln\left[1-\frac{C_m t}{M+m_o}\right] - gt$$

$$= \frac{F_o}{C_m}\ln\left[\frac{M+m_o}{M+m_o-C_m t}\right] - gt$$

2.1 2.8
2.2
2.3
2.5

$$x(t)=\int_0^t v(\tau)d\tau=\int_0^t\left\{\frac{F_0}{C_m}\ln\left[\frac{M+m_0}{M+m_0-C_m\tau}\right]-g\tau\right\}d\tau$$

ifadesinden bulunur.

2.8 ✓

Gazlı sistem:

$$d\rho=\frac{\rho}{np}dp \quad (n=1.4,\ izotermal)$$

$$\rho=\frac{M}{V} \qquad d\rho=-\frac{M}{V^2}dV=\frac{M}{V^2}Adx$$

$$dF=A\,dp \qquad dp=\frac{1}{A}dF$$

$$\frac{M}{V^2}Adx=\frac{M/V}{np}\,\frac{1}{A}dF$$

$$dF=\frac{A^2np}{V}dx=\frac{A^2np}{AL}dx$$

$$dF=\frac{nAp}{L}dx \qquad k_e=\frac{nAp}{L}$$

Nonlineer (Yay sabiti p'ye bağlı.)

$p\cong 1\,bar=10^5\,N/m^2$ için

$$k_e=\frac{1.4\times10^{-4}m^2\times10^5N/m^2}{0.15m}=93.3\ N/m$$

Sıvılı sistem:

$$\frac{dp}{\beta}=\frac{d\rho}{\rho} \qquad dp=\frac{dF}{A}$$

$$d\rho=\frac{M}{V^2}Adx \qquad \rho=\frac{M}{V}$$

$$\frac{dF}{A\beta}=\frac{M}{V^2}Adx.\frac{V}{M}$$

$$dF=\frac{A^2\beta}{V}dx=\frac{A^2\beta}{AL}dx$$

$$dF=\frac{\beta A}{L}dx \qquad k_e=\frac{\beta A}{L}$$

$$k_e=\frac{15000\times10^5N/m^2\times10^{-4}m^2}{0.15m}$$

$$=10^6\ N/m\ (lineer)$$

Metal için:

$$E=\frac{F/A}{x/L} \qquad F=\frac{EA}{L}x \qquad k_e=\frac{EA}{L}$$

$$k_e=\frac{200\times10^9N/m^2\times10^{-4}m^2}{0.15m}$$

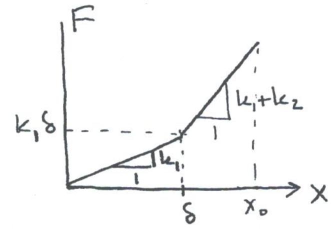
$$=1.33\times10^8\ N/m\ (lineer)$$

2.9 ✓

a) $k_e=k_1+k_2$ b) $k_e=\dfrac{k_1k_2}{k_1+k_2}$

c) $k_e=k_1+k'+k_4 \qquad k'=\dfrac{k_2k_3}{k_2+k_3}$

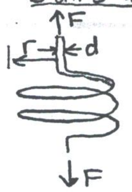
2.10



$$E_p=\frac{1}{2}k_1x_0^2+\frac{1}{2}k_2(x_0-\delta)^2$$

2.11

Daire kesitli dairesel helis yay:



$$M_b=Fr \qquad \frac{d\phi}{dz}=\frac{Fr}{GI_z}$$

(z tel boyunca ölçülüyor.)

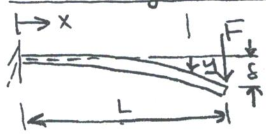
$$d\delta=r\,d\phi=\frac{Fr^2}{GI_z}dz$$

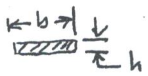
$$\delta=\int_0^{2\pi rn}\frac{Fr^2}{GI_z}dz \qquad I_z=\frac{\pi d^4}{32}$$

$$\delta=\int_0^{2\pi rn}\frac{32Fr^2}{G\pi d^4}dz=\frac{F32r^2}{G\cancel{\pi}d^4}2\cancel{\pi}rn$$

$$\delta=\frac{64r^3n}{Gd^4}F=\frac{1}{k_e}F \qquad k_e=\frac{Gd^4}{64r^3n}$$

Dikdörtgen kesitli plaka:





$$EI\frac{d^2y}{dx^2}=F(L-x)$$

x=0 da $y=0$, $\dfrac{dy}{dz}=0$

$$\frac{dy}{dx}=\frac{F}{EI}\left(Lx-\frac{x^2}{2}\right)+c_1$$

$$y=\frac{F}{EI}\left(L\frac{x^2}{2}-\frac{x^3}{6}\right)+c_1x+c_2$$

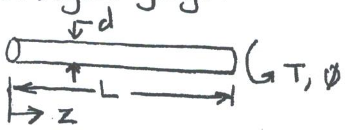
Sınır şartlarından $c_1=0$ $c_2=0$

$$\delta=y(L)=\frac{F}{EI}\left(\frac{L^3}{2}-\frac{L^3}{6}\right)=\frac{FL^3}{3EI}$$

$$F=k_e\delta \qquad k_e=\frac{3EI}{L^3} \qquad I=\frac{bh^3}{12}$$

$$k_e=\frac{bh^3E}{4L^3}$$

Torsiyon yayı:

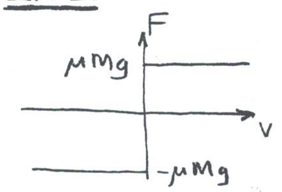


$$\frac{d\phi}{dz}=\frac{M_t}{G I_z} \qquad I_z=\frac{\pi d^4}{32} \qquad M_t=T$$

$$d\phi=\frac{T}{G I_z}dz \qquad \phi=\int_0^L \frac{32T}{G\pi d^4}dz$$

$$\phi=\frac{32LT}{\pi d^4 G}=\frac{1}{k_t}T \qquad k_t=\frac{\pi d^4 G}{32L}$$

2.12

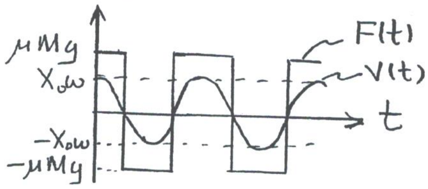


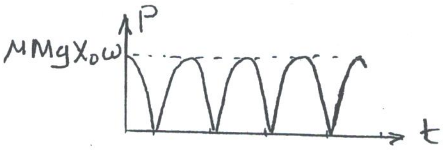
$$P=Fv=\mu MgV$$

$$x(t)=X_0\sin\omega t$$

$$V(t)=X_0\omega\cos\omega t$$

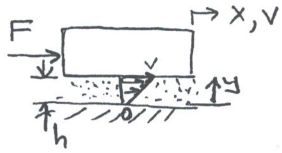
$$P=|\mu MgX_0\omega\cos\omega t|$$





2.13

$$\mu=\frac{\tau}{\frac{dV}{dy}}$$



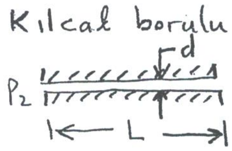
$$\mu=\frac{\tau . A}{\frac{V}{h}.A}=\frac{F}{\frac{VA}{h}} \qquad F=V\frac{\mu A}{h}$$

$$b=\frac{\mu A}{h}=\frac{0.03\ N.s/m^2\times 0.04m^2}{0.0001\ m}$$

$$b=12\ N.s/m$$

2.14

Kılcal borulu sönümleyici:



$$P_2-P_1=\frac{128\mu L}{\pi d^4}Q$$

$$\frac{F}{A}=\frac{128\mu L}{\pi d^4}AV$$

Piston alanı: A

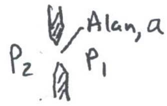
$$F=\left[\frac{128\mu LA^2}{\pi d^4}\right]V=bV$$

$$b=\frac{128\mu LA^2}{\pi d^4}$$

b sabit ve eleman lineer.

Orifisli sönümleyici:

$$Q=C_d a\sqrt{\frac{2}{\rho}(P_2-P_1)}$$



$$AV=C_d a\sqrt{\frac{2}{\rho}\frac{F}{A}}$$

$$F=\frac{A^3\rho}{2C_d^2a^2}V^2$$

(Nonlineer eleman.)

2.15

Orifis alanı: $A(x)=\pi R^2-\pi\left(r+\frac{a}{L}x\right)^2$

$$Q=C_d A(x)\sqrt{\frac{2}{\rho}\Delta p}$$

$$A_pV=C_dA(x)\sqrt{\frac{2}{\rho}\frac{F}{A_p}} \qquad (A_p\text{: Piston alanı})$$

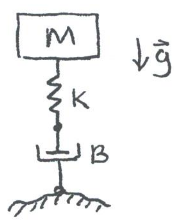
$$F=\frac{A_p^3\rho}{2C_d^2[A(x)]^2}V^2 \qquad \text{(Nonlineer eleman)}$$

$F=F_0$ (sabit) ise,

$$F_0=\frac{A_p^3\rho}{2C_d^2\left[\pi R^2-\pi\left(r+\frac{a}{L}x\right)^2\right]^2}V^2$$

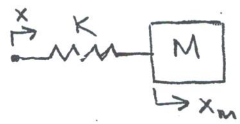
$$V=C_d\left[\pi R^2-\pi\left(r+\frac{a}{L}x\right)^2\right]\sqrt{\frac{2F_0}{A_p^3\rho}}$$

2.16



Bu askı sistemi taşıtın statik ağırlığını taşıyamaz.

2.17



$$x(t)=X_0\sin\omega t$$

Sistemin kütle gibi davranması durumunda: $|x|>>|x-x_m|$ ve $x\approx x_m$.

$$F = K(x-x_m) = M\frac{d^2x_m}{dt^2} \cong M\frac{d^2x}{dt^2}$$

ya da

$$x - x_m = \frac{M}{K}\frac{d^2x}{dt^2} = -\frac{M}{K}\omega^2 x_o \sin\omega t$$

Ama, $x >> |x - x_m|$ olduğundan

$$x_o \sin\omega t >> \frac{M\omega^2}{K} x_o \sin\omega t$$

ya da

$$\omega << \sqrt{\frac{K}{M}} \; ; \; \omega << \omega_n$$

Sistemin yay gibi davranması durumunda, x_m çok küçüktür.

$$|x| >> |x_m| \quad ve \quad |x| \cong |x - x_m|$$

$$F = K(x - x_m) = Kx = M\frac{d^2x_m}{dt^2}$$

$$\frac{d^2x_m}{dt^2} \cong \frac{K}{M} x_o \sin\omega t$$

$$x_m = -\frac{K}{M} x_o \frac{1}{\omega^2}\sin\omega t$$

Ama $|x| >> |x_m|$'den,

$$x_o \sin\omega t >> \frac{K}{M} x_o \frac{1}{\omega^2}\sin\omega t$$

$$\omega^2 > \frac{K}{M} \qquad \omega >> \sqrt{\frac{K}{M}} \qquad \omega >> \omega_n$$

Not: Kütlenin ivmesi yüksek frekanslarda ω^2 ile orantılı olup, bu ivmenin sağlanması çok büyük kuvvetler gerektirir. Kütleye uygulanan kuvvet sınırlı olduğundan kütlenin hareket genliği frekans arttıkça azalır.

2.18

$$F(t) = F_o \sin\omega t$$

Kütle: $a = \frac{F_o}{m}\sin\omega t \quad v = \left|\frac{-F_o}{m\omega}\right|\cos\omega t$

$$x = \left|\frac{-F_o}{m\omega^2}\right|\sin\omega t$$

ω arttıkça x genliği azalır. (2.17'deki açıklamaya bakın.)

Yay: $x = \frac{F_o}{k}\sin\omega t \quad v = \left|\frac{F_o\omega}{k}\right|\cos\omega t$

$$a = \left|\frac{-F_o\omega^2}{k}\right|\sin\omega t$$

ω arttıkça x genliği değişmez zira elemanda eylemsizlik yoktur.

Sönümleyici:

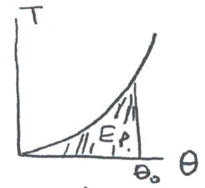
$$v = \frac{F_o}{b}\sin\omega t \qquad x = \left|\frac{-F_o}{b\omega}\right|\cos\omega t$$

$$a = \left|\frac{-F_o\omega}{b}\right|\cos\omega t$$

Elemanın eylemsizliği yok, ω arttıkça x'in genliği azalır, a'nın genliği artar.

Dönel elemanlar için bu elemanların değişkenleri cinsinden yazılmış benzer ifadeler ve yorumlar geçerlidir.

2.19



$$T = k\theta + h\theta^3$$

$$E_p = \int_0^{\theta_o} T\,d\theta$$

$$E_p = \int_0^{\theta_o}(k\theta + h\theta^3)\,d\theta = \frac{k}{2}\theta_o^2 + \frac{h}{4}\theta_o^4$$

2.20

a) $k_{te} = \frac{k_{t_1}k_{t_2}}{k_{t_1} + k_{t_2}}$ b) $k_{te} = k_{t_1} + k_{t_2}$

2.21

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\tau}{\frac{\omega r}{\varepsilon}} \qquad \tau = \mu\frac{\omega r}{\varepsilon}$$

Teğetsel kuvvet $= \tau 2\pi r a$

$$T = (2\pi r \tau a) r = 2\pi r^2 a \tau$$

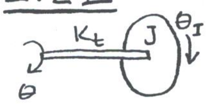
$$T = \frac{2\pi r^3 a\mu}{\varepsilon}\omega = b_{te}\omega$$

$$b_{te} = \frac{2\pi r^3 a\mu}{\varepsilon}$$

F_o için $T = F_o R$

$$\omega_o = \frac{F_o R \varepsilon}{2\pi r^3 a\mu}$$

2.22



$\theta(t) = \theta_0 \sin \omega t$

Not: Çözüm ve yorumlar için 2.17 ile paralellik kurun.

2.23

Verilen referans yönleri esas alınırsa,

a) $v_2 = r\omega$, $f_2 = -\frac{1}{r} T_1$

b) $v_1 = -2v_2$, $f_1 = \frac{1}{2} f_2$

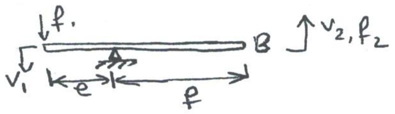
c) $v_2 = r\omega$, $f_2 = -\frac{1}{r} T_1$

d) $v_1 = -\frac{a}{b} v_2$, $f_1 = \frac{b}{a} f_2$

e) $\omega_1 = \frac{n_2}{n_1}\omega_2$, $T_1 = -\frac{n_1}{n_2} T_2$

f) $v_2 = \tan\theta \, v_1$, $f_2 = -\frac{1}{\tan\theta} f_1$

2.24



$v_1 = \frac{e}{\ell} v_2$

$f_1 = -\frac{\ell}{e} f_2$

B'de yay varsa,

$$x_1 = \frac{e}{\ell} x_2 \qquad f_1 = -\frac{\ell}{e} f_2 = +\frac{\ell}{e} f_k$$

$$f_1 = \frac{\ell}{e} k x_2 = \frac{\ell^2}{e^2} k x_1 = k_e x_1$$

$$k_e = \left(\frac{\ell}{e}\right)^2 k$$

B'de sönümleyici varsa,

$$f_1 = -\frac{\ell}{e} f_2 = +\frac{\ell}{e} f_b = \frac{\ell}{e} b v_2$$

$$f_1 = \frac{\ell^2}{e^2} b v_1 = b_e v_1 \qquad b_e = \left(\frac{\ell}{e}\right)^2 b$$

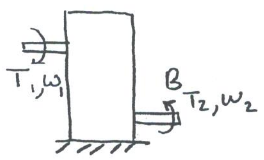
B'de kütle varsa,

$$f_1 = -\frac{\ell}{e} f_2 = +\frac{\ell}{e} f_m = \frac{\ell}{e} m \ddot{x}_2$$

$$f_1 = \left(\frac{\ell}{e}\right)^2 m \ddot{x}_1 = m_e \ddot{x}_1$$

$$m_e = \left(\frac{\ell}{e}\right)^2 m$$

2.25



$$\omega_1 = \frac{n_2}{n_1}\omega_2$$

$$T_1 = -\frac{n_1}{n_2} T_2$$

B'de yay varsa,

$$\theta_1 = \frac{n_2}{n_1}\theta_2 \qquad T_1 = -\frac{n_1}{n_2} T_2 = \frac{n_1}{n_2} T_k$$

$$T_1 = \frac{n_1}{n_2} k_t \theta_2 = \left(\frac{n_1}{n_2}\right)^2 \theta_1 = k_{te}\theta_1$$

$$k_{te} = \left(\frac{n_1}{n_2}\right)^2 k_t$$

B'de sönümleyici varsa,

$$T_1 = -\frac{n_1}{n_2} T_2 = \frac{n_1}{n_2} T_b = \frac{n_1}{n_2} b_t \omega_2$$

$$T_1 = \left(\frac{n_1}{n_2}\right)^2 b_t \omega_1 = b_{te}\omega_1$$

$$b_{te} = \left(\frac{n_1}{n_2}\right)^2 b_t$$

B'de eylemsizlik varsa,

$$T_1 = -\frac{n_1}{n_2} T_2 = \frac{n_1}{n_2} T_I = \frac{n_1}{n_2} I \ddot{\theta}_2$$

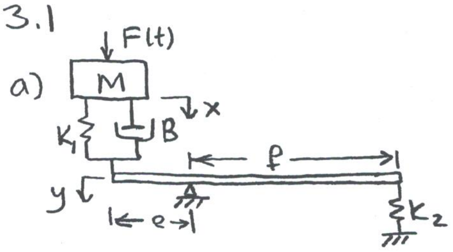
$$T_1 = \left(\frac{n_1}{n_2}\right)^2 I \ddot{\theta}_1 = I_e \ddot{\theta}_1$$

$$I_e = \left(\frac{n_1}{n_2}\right)^2 I$$

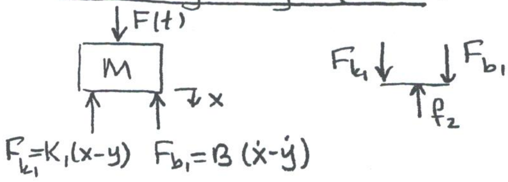
BÖLÜM 3

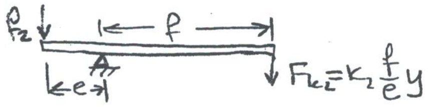
3.1

a)



Newton yasasıyla çözüm:





$f K_2 \frac{f}{e} y - f_2 e = 0$ (1) $f_2 - F_{k_1} - F_{b_1} = 0$ (2)

$M\ddot{x} = F(t) - K_1(x-y) - B(\dot{x}-\dot{y})$ (3)

ya da, $f_2 = K_2 \frac{f^2}{e^2} y$

$K_2\left(\frac{f}{e}\right)^2 y - K_1(x-y) - B(\dot{x}-\dot{y}) = 0$ $s \equiv \frac{d}{dt}$

$$y = \frac{Bs+K_1}{Bs+K_1+K_2\left(\frac{f}{e}\right)^2}\, x$$

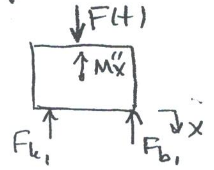
$$x-y = \frac{K_2\left(\frac{f}{e}\right)^2}{Bs+K_1+K_2\left(\frac{f}{e}\right)^2}\, x$$, (3)'de kullanılırsa,

$$Ms^2x = F(t) - \frac{(Bs+K_1)K_2\left(\frac{f}{e}\right)^2}{Bs+\underbrace{K_1+K_2\left(\frac{f}{e}\right)^2}_{\bar{K}}}\, x$$

$$MB\dddot{x} + \bar{K}M\ddot{x} + BK_2\left(\frac{f}{e}\right)^2\dot{x} + K_1K_2\left(\frac{f}{e}\right)^2 x = B\dot{F}(t) + \bar{K}F(t)$$

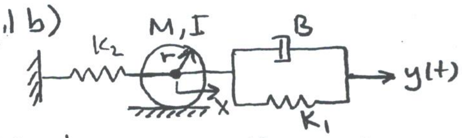
Dinamik denge metodu:

Kütlenin serbest cisim diyagramı ve denklem(3) aşağıdaki gibi olur. Diğer herşey yukarıdaki gibidir.

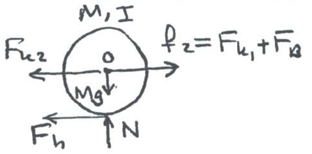


$F(t) - M\ddot{x} - K_1(x-y) - B(\dot{x}-\dot{y}) = 0$

3.1 b)



Newton yasası ile çözüm:



$F_{k_1} = K_1(y-x)$

$F_B = B(\dot{y}-\dot{x})$

$F_{k_2} = K_2 x$

$M\ddot{x} = F_{k_1} + F_B - F_{k_2} - F_h$

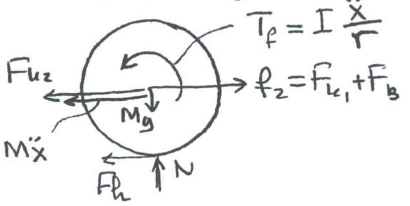
$I\frac{\ddot{x}}{r} = F_h . r$ $F_h = \frac{I}{r^2}\ddot{x}$ (I ağırlık merkezine göredir.)

ya da

$M\ddot{x} = K_1(y-x) + B(\dot{y}-\dot{x}) - K_2x - \frac{I}{r^2}\ddot{x}$

$$\left(M+\frac{I}{r^2}\right)\ddot{x} + B\dot{x} + (K_1+K_2)x = B\dot{y}(t) + K_1y(t)$$

Dinamik denge metodu:



$-F_{k_2} - M\ddot{x} + F_{k_1} + F_B - F_h = 0$

$F_h . r - I\frac{\ddot{x}}{r} = 0$

Sonuç yukarıdaki gibidir.

Enerji metodu:

$E_{top} = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{r}\right)^2 + \frac{1}{2}K_2x^2$

$P = F_B\dot{x} + F_k\dot{x}$

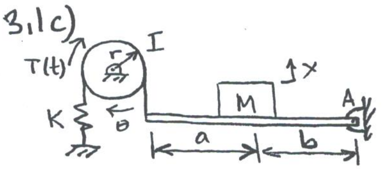
$= B(\dot{y}-\dot{x})\dot{x} + K_1(y-x)\dot{x}$

$\frac{dE_{top}}{dt} = P$

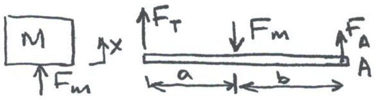
$M\dot{x}\ddot{x} + \left(\frac{I}{r^2}\right)\dot{x}\ddot{x} + K_2x\dot{x} = B(\dot{y}-\dot{x})\dot{x} + K_1(y-x)\dot{x}$

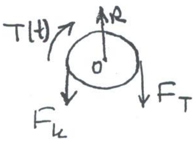
$$\left(M+\frac{I}{r^2}\right)\ddot{x} + B\dot{x} + (K_1+K_2)x = B\dot{y}(t) + K_1y(t)$$

3.1c)



Newton yasası:





$$M\ddot{x}=F_M$$
$$\sum T_A=0=F_T(a+b)-F_M b$$
$$\sum T_o=I\ddot{\theta}=F_T r-F_k r+T(t)$$

$$F_k=kr\theta \qquad \theta r=-\frac{a+b}{b}x$$

Ya da,

$$F_T=\frac{I}{r}\ddot{\theta}+F_k-\frac{1}{r}T(t)=-\frac{I}{r}\left(\frac{a+b}{b}\right)\frac{\ddot{x}}{r}-kr\left(\frac{a+b}{b}\right)\frac{x}{r}-\frac{1}{r}T(t)$$

$$F_M=\left(\frac{a+b}{b}\right)F_T=-\frac{I}{r^2}\left(\frac{a+b}{b}\right)^2\ddot{x}-k\left(\frac{a+b}{b}\right)^2x-\frac{(a+b)}{br}T(t)$$

$$\left[M+\frac{I}{r^2}\left(\frac{a+b}{b}\right)^2\right]\ddot{x}+k\left(\frac{a+b}{b}\right)^2x=-\frac{a+b}{br}T(t)$$

Enerji metodu:

$$E_T=\frac{1}{2}I\dot{\theta}^2+\frac{1}{2}M\dot{x}^2+\frac{1}{2}K(r\theta)^2$$

$$\mathbb{P}=T(t)\dot{\theta} \qquad \theta=-\left(\frac{a+b}{br}\right)x$$

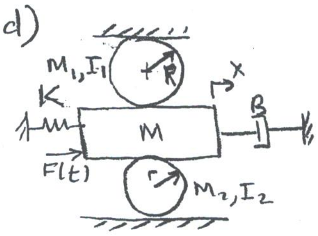
$$E_T=\frac{1}{2}\frac{I}{r^2}\left(\frac{a+b}{b}\right)^2\dot{x}^2+\frac{1}{2}M\dot{x}^2+\frac{1}{2}K\left(\frac{a+b}{b}\right)^2x^2$$

$$\frac{dE_T}{dt}=\mathbb{P} \qquad \mathbb{P}=-T(t)\left(\frac{a+b}{br}\right)\dot{x}$$

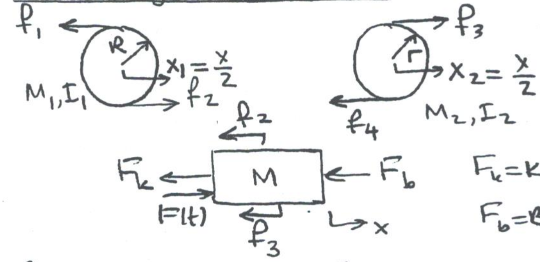
$$\frac{I}{r^2}\left(\frac{a+b}{b}\right)^2\dot{x}\ddot{x}+M\dot{x}\ddot{x}+K\left(\frac{a+b}{b}\right)^2x\dot{x}=-T(t)\left(\frac{a+b}{br}\right)\dot{x}$$

$$\left[M+\frac{I}{r^2}\left(\frac{a+b}{b}\right)^2\right]\ddot{x}+K\left(\frac{a+b}{b}\right)^2x=-\left(\frac{a+b}{br}\right)T(t)$$

3.1d)



Newton yasasıyla çözüm:



$F_k=Kx$

$F_b=B\dot{x}$

$$f_1R+f_2R=I_1\alpha_1=I_1\frac{\ddot{x}}{2R} \qquad (1)$$
$$f_2-f_1=M_1\ddot{x}_1=M_1\frac{\ddot{x}}{2} \qquad (2)$$
$$f_3r+f_4r=I_2\alpha_2=I_2\frac{\ddot{x}}{2r} \qquad (3)$$
$$f_3-f_4=M_2\ddot{x}_2=M_2\frac{\ddot{x}}{2} \qquad (4)$$
$$M\ddot{x}=-f_2-f_3-F_b-F_k+F(t)=-f_2-f_3-B\dot{x}-Kx+F(t) \qquad (5)$$

(1) ve (2)den

$$f_2=\left(\frac{M_1}{4}+\frac{I_1}{4R^2}\right)\ddot{x}$$

(3) ve (4) den

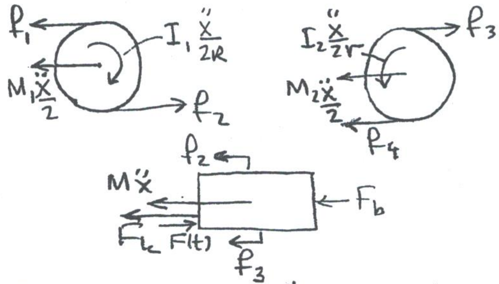
$$f_3=\left(\frac{M_2}{4}+\frac{I_2}{4r^2}\right)\ddot{x}$$

$$M\ddot{x}=-\left(\frac{M_1}{4}+\frac{I_1}{4R^2}\right)\ddot{x}-\left(\frac{M_2}{4}+\frac{I_2}{4r^2}\right)\ddot{x}-B\dot{x}-Kx+F(t)$$

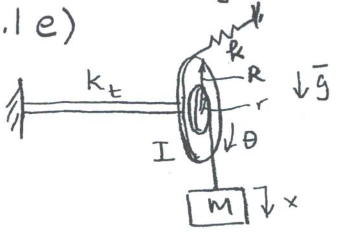
$$\left[M+\frac{M_1}{4}+\frac{M_2}{4}+\frac{I_1}{4R^2}+\frac{I_2}{4r^2}\right]\ddot{x}+B\dot{x}+Kx=F(t)$$

Dinamik denge metodu:

Serbest cisim diyagramları aşağıdaki gibi olup, statik kuvvet ve moment denge denklemlerinden aynı sonuç bulunur.

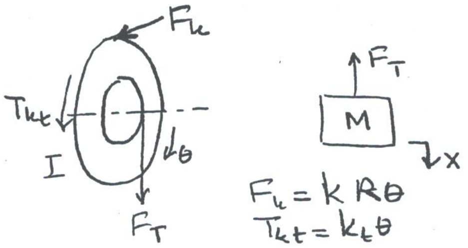


3.1e)



Yerçekimi sistemi denge konumuna geri getirmeye çalışmadığından, koordinatlar denge konumundan ölçülürse yerçekimi dikkate alınmaz.

Newton yasasıyla çözüm:



$F_k = kR\theta$

$T_{kt} = k_t\theta$

$$I\ddot\theta = -F_kR + F_Tr - T_{kT}$$

$$M\ddot x = -F_T \qquad x = \theta r$$

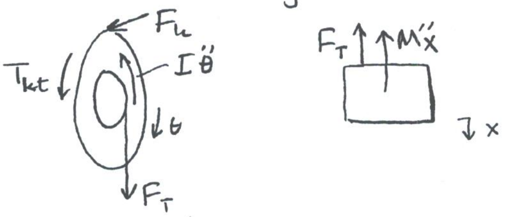
ya da,

$$I\ddot\theta = -(kR\theta)R - (Mr\ddot\theta)r - k_t\theta$$

$$(I + Mr^2)\ddot\theta + (k_t + kR^2)\theta = 0$$

Dinamik denge metodu:

Serbest cisim diyagramları aşağıdaki gibi olup, statik kuvvet ve moment denge denklemlerinden aynı sonuç bulunur.



Enerji metodu:

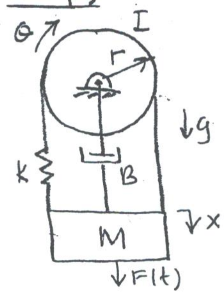
$$E_T = \frac{1}{2}k_t\theta^2 + \frac{1}{2}k(R\theta)^2 + \frac{1}{2}M(r\dot\theta)^2 + \frac{1}{2}I\dot\theta^2$$

$$P = 0$$

$$\frac{dE_T}{dt} = k_t\theta\dot\theta + kR^2\theta\dot\theta + Mr^2\dot\theta\ddot\theta + I\dot\theta\ddot\theta = 0$$

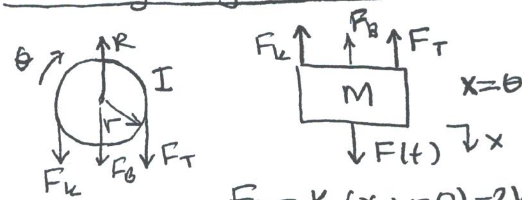
$$(I + Mr^2)\ddot\theta + (k_t + kR^2)\theta = 0$$

3.1f)



Koordinatlar statik denge konumundan ölçülürse, yerçekimi dikkate alınmaz.

Newton yasasıyla çözüm:



$x = \theta r$

$F_k = K(x + r\theta) = 2Kx$

$F_B = B\dot x$

$$I\ddot\theta = F_Tr - F_kr \qquad \frac{I}{r}\ddot x = F_Tr - F_kr$$

$$M\ddot x = F(t) - F_k - F_T - F_B$$

$$\frac{I}{r}\ddot x = -Mr\ddot x - rF_k + rF(t) - rF_k - rF_B$$

$$\left(M + \frac{I}{r^2}\right)\ddot x + B\dot x + 2K(2x) = F(t)$$

$$\left(M + \frac{I}{r^2}\right)\ddot x + B\dot x + 4Kx = F(t)$$

Enerji metodu:

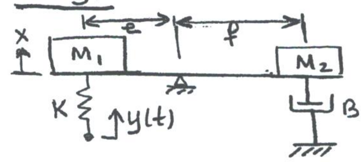
$$E_T = \frac{1}{2}I\left(\frac{\dot x}{r}\right)^2 + \frac{1}{2}M\dot x^2 + \frac{1}{2}K(2x)^2$$

$$P = F(t)\dot x - B\dot x^2$$

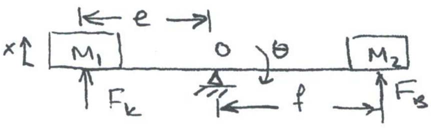
$$\frac{dE_T}{dt} = \frac{I}{r^2}\dot x\ddot x + M\dot x\ddot x + 4Kx\dot x = F(t)\dot x - B\dot x^2$$

$$\left(M + \frac{I}{r^2}\right)\ddot x + B\dot x + 4Kx = F(t)$$

3.1g)



Newton yasasıyla çözüm:



$\theta e = x$

$$I_0 = M_1e^2 + M_2f^2 \qquad F_k = K(y - x)$$

$$\Sigma T_0 = I_0\ddot\theta = +eF_k - fF_B \qquad F_B = B\left(\frac{f}{e}\right)\dot x$$

$$(M_1e^2 + M_2f^2)\frac{\ddot x}{e} = +eK(y - x) - fB\left(\frac{f}{e}\right)\dot x$$

$$\left[M_1 + \left(\frac{f}{e}\right)^2 M_2\right]\ddot x + \left(\frac{f}{e}\right)^2 B\dot x + Kx = Ky(t)$$

Dinamik Denge metodu:

Yukarıdaki serbest cisim diyagramına $-\theta$ yönünde $I_0\ddot\theta$ kadar D'Alambert momenti eklenip, statik moment dengesinden aynı sonuç bulunur.

Enerji metodu:

$$E_T = \frac{1}{2}M_1\dot{x}^2 + \frac{1}{2}M_2\left(\frac{f}{e}\dot{x}\right)^2 + \frac{1}{2}K(x-y)^2$$

$$\mathbb{P} = -F_b\left(\frac{f}{e}\dot{x}\right) + F_k\,\dot{y}$$

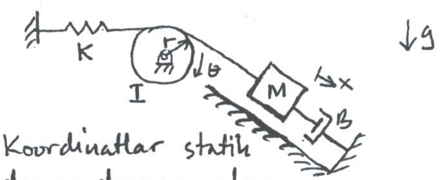
$$\frac{dE_t}{dt} = M_1\dot{x}\ddot{x} + M_2\left(\frac{f}{e}\right)^2\dot{x}\ddot{x} + K(x-y)\dot{x} - k(x-y)\dot{y}$$

$$\frac{dE_T}{dt} = \mathbb{P}$$

$$M_1\dot{x}\ddot{x} + M_2\left(\frac{f}{e}\right)^2\dot{x}\ddot{x} + k(x-y)\dot{x} - k\cancel{(x-y)}\dot{y} = -B\left(\frac{f}{e}\right)^2\dot{x}^2 + k\cancel{(y-x)}\dot{y}$$

$$\left[M_1 + M_2\left(\frac{f}{e}\right)^2\right]\ddot{x} + B\left(\frac{f}{e}\right)^2\dot{x} + Kx = Ky(t)$$

3.1 h)



Koordinatlar statik denge durumundan ölçülürse yerçekimi dikkate alınmaz.

Newton yasasıyla çözüm:

$F_k = Kx$ $\qquad$ $F_B = B\dot{x}$ $\qquad$ $x = \theta r$

$$I\ddot{\theta} = (F_T - F_k)r \quad \text{ya da} \quad I\frac{\ddot{x}}{r^2} = F_T - Kx$$

$$M\ddot{x} = -F_T - F_B \quad \text{ya da} \quad M\ddot{x} = -F_T - B\dot{x}$$

$$M\ddot{x} = -I\frac{\ddot{x}}{r^2} - Kx - B\dot{x}$$

$$\left(M + \frac{I}{r^2}\right)\ddot{x} + B\dot{x} + Kx = 0$$

Dinamik denge metodu:

Yukarıda soldaki serbest cisim diyagramına $-\theta$ yönünde $I\ddot{\theta}$ kadar D'Alambert momenti; sağdakine de $-x$ yönünde $M\ddot{x}$ kadar D'Alambert kuvveti koyulup, statik kuvvet ve moment dengesi denklemleri yazılısa aynı sonuç elde edilir.

Enerji metodu:

$$E_T = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}M\dot{x}^2 + \frac{1}{2}Kx^2 \qquad x = \theta r$$

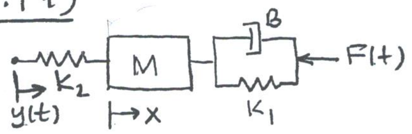
$$\mathbb{P} = -B\dot{x}^2 \qquad \frac{dE_T}{dt} = \mathbb{P}$$

$$\frac{d}{dt}\left[\frac{1}{2}\frac{I}{r^2}\dot{x}^2 + \frac{1}{2}M\dot{x}^2 + \frac{1}{2}Kx^2\right] = -B\dot{x}^2$$

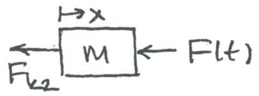
$$\frac{I}{r^2}\dot{x}\ddot{x} + M\dot{x}\ddot{x} + Kx\dot{x} = -B\dot{x}^2$$

$$\left(M + \frac{I}{r^2}\right)\ddot{x} + B\dot{x} + Kx = 0$$

3.1 i)



Newton yasasıyla çözüm:



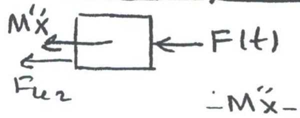
Not: Sağ taraftaki K_1 yayı ve B sönümleyicisi kütleleri olmadığından F(t) kuvvetini aynen kütleye iletirler. Yani kütlenin dinamik davranışı üzerine bu elemanların bir etkisi yoktur.

$$M\ddot{x} = -F(t) - F_{k2} \qquad F_{k2} = K_2(x-y)$$

$$M\ddot{x} = -K_2(x-y) - F(t)$$

$$M\ddot{x} + K_2 x = K_2 y(t) - F(t)$$

Dinamik denge metodu:

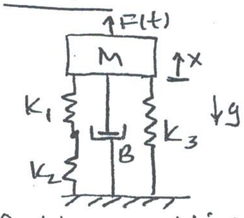


$$-M\ddot{x} - F(t) - F_{k2} = 0$$

$$-M\ddot{x} - K_2(x-y) - F(t) = 0$$

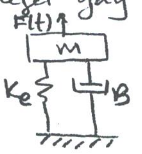
$$M\ddot{x} + K_2 x = K_2 y(t) - F(t)$$

3.1 j)



Koordinatlar denge durumundan ölçülürse yerçekimi dikkate alınmaz.

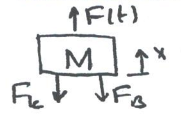
Problem verildiği haliyle de çözülebilir. Ancak işlemleri kolaylaştırmak için eşdeğer yay sabiti bulunursa,



$$K' = \frac{K_1 K_2}{K_1 + K_2}$$

$$K_e = K' + K_3$$

Newton yasasıyla çözüm:

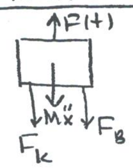


$F_k = K_e x$ $\quad F_B = B\dot{x}$

$M\ddot{x} = F(t) - F_k - F_B$

$M\ddot{x} + B\dot{x} + K_e x = F(t)$

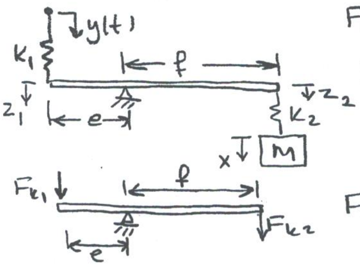
Dinamik denge metodu:



$F(t) - M\ddot{x} - K_e x - B\dot{x} = 0$

$M\ddot{x} + B\dot{x} + K_e x = F(t)$

3.1k)



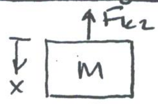
$F_{k_1} = K_1(y - z_1)$

$F_{k_2} = K_2(x - z_2)$

$z_2 = -\frac{\ell}{e} z_1$

$F_{k_2}\ell - F_{k_1} e = 0$

Newton yasası:



$M\ddot{x} = -F_{k_2} = -k_2(x - z_2)$

Ama, $k_2\ell(x - z_2) - k_1 e(y - z_1) = 0$

$k_2\ell x - k_2\ell z_2 - k_1 e y + k_1 e\left(-\frac{e}{\ell}\right) z_2 = 0$

$\left(k_2\ell + k_1\frac{e^2}{\ell}\right) z_2 = k_2\ell x - k_1 e y$

$\left(k_2 + k_1\frac{e^2}{\ell^2}\right) z_2 = k_2 x - k_1\frac{e}{\ell} y$

$$z_2 = \frac{k_2}{k_2 + k_1\frac{e^2}{\ell^2}} x - \frac{k_1\frac{e}{\ell}}{k_2 + k_1\frac{e^2}{\ell^2}} y$$

$$M\ddot{x} + k_2 x = \frac{k_2^2}{k_2 + k_1\frac{e^2}{\ell^2}} x - \frac{k_1 k_2\frac{e}{\ell}}{k_2 + k_1\frac{e^2}{\ell^2}} y(t)$$

$$M\ddot{x} + \left(k_2 - \frac{k_2^2}{k_2 + k_1\frac{e^2}{\ell^2}}\right) x = -\frac{k_1 k_2\frac{e}{\ell}}{k_2 + k_1\frac{e^2}{\ell^2}} y(t)$$

$$M\ddot{x} + \left(\frac{k_1 k_2\frac{e^2}{\ell^2}}{k_2 + k_1\frac{e^2}{\ell^2}}\right) x = -\frac{k_1 k_2\frac{e}{\ell}}{k_2 + k_1\frac{e^2}{\ell^2}} y(t)$$

Dinamik denge metodu:

Kütlenin serbest cisim diyagramına $-x$ yönünde $M\ddot{x}$ kadar bir D'Alambert kuvveti eklenip, statik kuvvet dengesi yazılırsa, aynı sonuç bulunur.

Enerji metodu:

$E_T = \frac{1}{2}K_1(y - z_1)^2 + \frac{1}{2}k_2(z_2 - x)^2 + \frac{1}{2}M\dot{x}^2$

$P = \dot{y}F_{k_1} = K_1(y - z_1)\dot{y}$ $\quad \frac{dE_T}{dt} = P$

$K_1(y - z_1)(\cancel{\dot{y}} - \dot{z}_1) + K_2(z_2 - x)(\dot{z}_2 - \dot{x}) + M\dot{x}\ddot{x}$

$= \cancel{K_1(y - z_1)\dot{y}}$

$-F_{k_1}\left(-\frac{e}{\ell}\dot{z}_2\right) - F_{k_2}\dot{z}_2 + F_{k_2}\dot{x} + M\dot{x}\ddot{x} = 0$

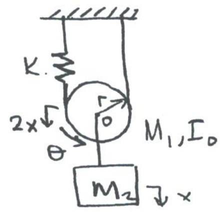
$\left(F_{k_1}\cancel{\frac{e}{\ell}} - \cancel{F_{k_2}}\right)\dot{z}_2 + F_{k_2}\cancel{\dot{x}} + M\cancel{\dot{x}}\ddot{x} = 0$

$M\ddot{x} + F_{k_2} = 0$

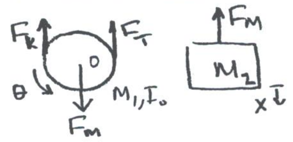
$M\ddot{x} + K_2(x - z_2) = 0$ $\quad z_2$ yerine koyulup terimler düzenlenirse,

$$M\ddot{x} + \frac{K_1 K_2\frac{e^2}{\ell^2}}{K_2 + K_1\frac{e^2}{\ell^2}} x = -\frac{K_1 K_2\frac{e}{\ell}}{K_2 + K_1\frac{e^2}{\ell^2}} y(t)$$

3.1 ℓ)



Newton Kanunu:



$F_k = 2Kx$ $\quad x = \theta r$

$I_o\ddot{\theta} = F_T r - F_k r$ ya da $I_o\frac{\ddot{x}}{r} = F_T r - F_k r$

$M_1\ddot{x} = F_M - F_T - F_k$ $\quad M_2\ddot{x} = -F_M$

$\left(M_1 + M_2 + \frac{I_o}{r^2}\right)\ddot{x} + 4Kx = 0$

Enerji metodu:

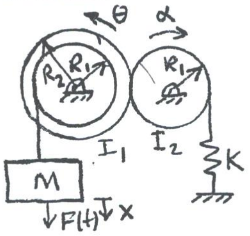
$E_T = \frac{1}{2}K(2x)^2 + \frac{1}{2}M_1\dot{x}^2 + \frac{1}{2}I_o\left(\frac{\dot{x}}{r}\right)^2 + \frac{1}{2}M_2\dot{x}^2$

$P = 0$ $\quad \frac{dE_T}{dt} = 0$

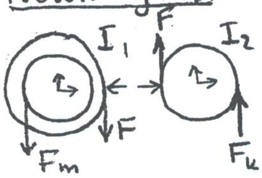
$4Kx\dot{x} + M_1\dot{x}\ddot{x} + \frac{I_o}{r^2}\dot{x}\ddot{x} + M_2\dot{x}\ddot{x} = 0$

$\left(M_1 + M_2 + \frac{I_o}{r^2}\right)\ddot{x} + 4Kx = 0$

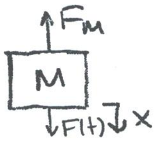
3.1 m)



Newton yasası:



$\theta R_1 = x \qquad \dfrac{\theta}{\alpha} = \dfrac{R_1}{R_2}$



$F_k = K\theta R_2 = K\dfrac{R_2}{R_1}x$

$I_1\left(\dfrac{\ddot{x}}{R_1}\right) = F_M R_1 - F R_2$

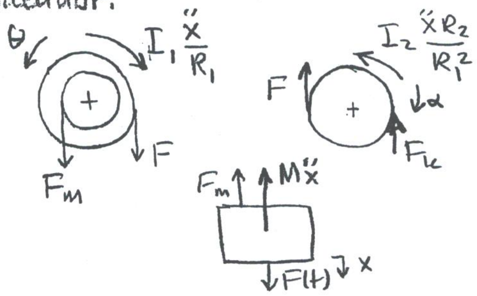
$I_2\left(\dfrac{\ddot{x}R_2}{R_1^2}\right) = FR_1 - F_k R_1 = FR_1 - KR_2 x$

$M\ddot{x} = -F_M + F(t)$ ya da,

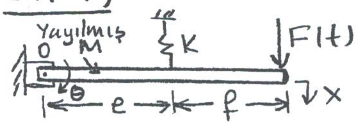
$$\left[M + \frac{I_2R_2^2}{R_1^4} + \frac{I_1}{R_1^2}\right]\ddot{x} + K\frac{R_2^2}{R_1^2}x = F(t)$$

Dinamik denge metodu:

Serbest cisim diyagramına aşağıdaki gibi D'Alambert kuvvet ve momentleri eklenip, statik kuvvet ve moment denklemleri yazılırsa aynı sonuç bulunur.



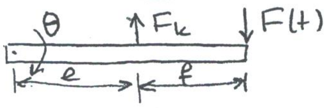
3.1 n)



$I_o = \frac{1}{3}M(e+f)^2$

$\dot{x} = \dot{\theta}(e+f)$

Newton yasası:



$F_k = K\dfrac{e}{(e+f)}x$

$I_o\ddot{\theta} = -eF_k + (e+f)F(t)$

$\frac{1}{3}M(e+f)^2\dfrac{\ddot{x}}{(e+f)} = -\dfrac{e^2}{e+f}Kx + (e+f)F(t)$

$$\frac{M}{3}\ddot{x} + \left(\frac{e}{e+f}\right)^2 Kx = F(t)$$

Dinamik denge metodu:

Serbest cisim diyagramına $-\theta$ yönünde $I_o\ddot{\theta}$ kadar D'Alambert momenti eklenip, statik moment dengesi yazılırsa aynı sonuç bulunur.

Enerji metodu:

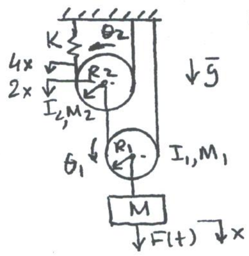
$E_T = \frac{1}{2}I_o\left(\dfrac{\dot{x}}{e+f}\right)^2 + \frac{1}{2}K\left(\dfrac{e}{e+f}x\right)^2$

$P = F(t)\dot{x} \qquad \dfrac{dE_T}{dt} = P$

$\dfrac{I_o}{(e+f)^2}\dot{x}\ddot{x} + K\left(\dfrac{e}{e+f}\right)^2 x\dot{x} = F(t)\dot{x}$

$$\frac{M}{3}\ddot{x} + \left(\frac{e}{e+f}\right)^2 Kx = F(t)$$

3.1 o)

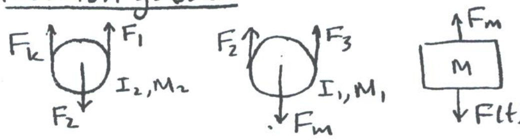


Koordinatların statik denge konumundan ölçüldüğü kabul edilirse, yerçekimi dikkate alınmaz.

$F_k = 4Kx$

$\theta_1 = \dfrac{x}{R_1} \qquad \theta_2 = \dfrac{2x}{R_2}$

Newton yasası:



$M_2(2\ddot{x}) = F_2 - F_k - F_1 \qquad I_2\left(\dfrac{2\ddot{x}}{R_2}\right) = (F_1 - F_k)R_2$

$M_1\ddot{x} = F_m - F_2 - F_3 \qquad I_1\dfrac{\ddot{x}}{R_1} = (F_3 - F_2)R_1$

$M\ddot{x} = F(t) - F_m$ ya da

$$\left(M + M_1 + 4M_2 + \frac{I_1}{R_1^2} + \frac{4I_2}{R_2^2}\right)\ddot{x} + 16Kx = F(t)$$

Enerji metodu:

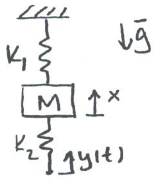
$E_T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}M_1\dot{x}^2 + \frac{1}{2}I_1\left(\dfrac{\dot{x}}{R_1}\right)^2 + \frac{1}{2}M_2(2\dot{x})^2 + \frac{1}{2}I_2\left(\dfrac{2\dot{x}}{R_2}\right)^2 + \frac{1}{2}K(4x)^2$

$P = F(t)\dot{x} = \dfrac{dE_T}{dt}$

$M\dot{x}\ddot{x} + M_1\dot{x}\ddot{x} + \dfrac{I_1}{R_1^2}\dot{x}\ddot{x} + 4M_2\dot{x}\ddot{x} + \dfrac{4I_2}{R_2^2}\dot{x}\ddot{x} + 16Kx\dot{x} = F(t)\dot{x}$

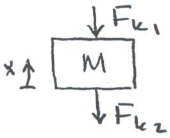
$$\left(M + M_1 + 4M_2 + \frac{I_1}{R_1^2} + \frac{4I_2}{R_2^2}\right)\ddot{x} + 16Kx = F(t)$$

3.1 p)



Koordinatlar statik denge durumundan itibaren tanımlansın. $\vec{g}$ sistemi bu duruma geri getirmeye çalışmadığından dikkate alınmaz.

Newton yasası:

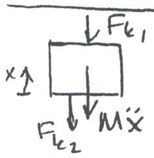


$$F_{k_1} = K_1 x$$
$$F_{k_2} = K_2 (x - y)$$

$$M\ddot{x} = -F_{k_1} - F_{k_2} = -K_1 x - K_2 (x - y)$$

$$M\ddot{x} + (K_1 + K_2)x = K_2 y(t)$$

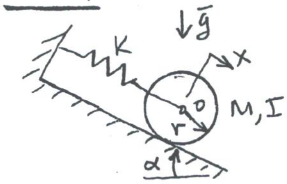
Dinamik denge metodu:



$$-M\ddot{x} - F_{k_1} - F_{k_2} = 0$$

alıp sonuç yukarıdaki gibidir.

3.1 r)



Koordinatlar statik denge durumundan ölçülsün. $\vec{g}$ sistemi bu duruma geri getirmeye çalışmadığından dikkate alınmaz.

Newton yasası:

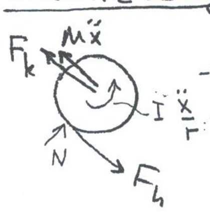
$$F_k = Kx \qquad x = \theta r$$
$$M\ddot{x} = -F_k + F_h = -Kx + F_h$$
$$I\frac{\ddot{x}}{r} = -F_h r$$

$$M\ddot{x} + Kx = F_h = -I\frac{\ddot{x}}{r^2}$$

$$\left(M + \frac{I}{r^2}\right)\ddot{x} + Kx = 0$$

(I eylemsizliği O'ya göredir.)

Dinamik denge metodu:



$$-M\ddot{x} - F_k + F_h = 0$$
$$-I\frac{\ddot{x}}{r} - F_h r = 0$$

Sonuç yukarıdaki gibi olur.

Enerji metodu:

$$E_T = \frac{1}{2}Kx^2 + \frac{1}{2}M\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{r}\right)^2$$

$$\frac{dE_T}{dt} = P = 0$$

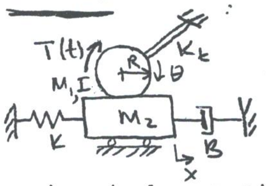
$$Kx\dot{x} + M\dot{x}\ddot{x} + \frac{I}{r^2}\dot{x}\ddot{x} = 0$$

$$\left(M + \frac{I}{r^2}\right)\ddot{x} + Kx = 0$$

3.1 s)

Denklemler ve sonuç 3.1r'deki gibidir.

3.1 t)



Newton yasası:

Not: Tekerin ağırlık merkezi değişmediğinden sistemin dinamiğine M_1'in katkısı yoktur.

$$F_k = Kx \qquad F_b = B\dot{x} \qquad T_k = K_t\theta \qquad x = -\theta R$$

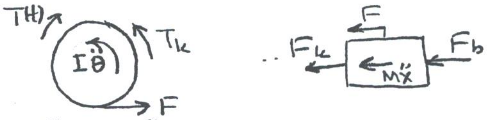
$$-I\frac{\ddot{x}}{R} = T(t) - T_k - FR = T(t) + K_t\frac{x}{R} - FR$$

$$M_2\ddot{x} = -F_k - F_b - F = -Kx - B\dot{x} - F$$

$$M_2\ddot{x} + B\dot{x} + Kx = -F = -\frac{I}{R^2}\ddot{x} - K_t\frac{x}{R^2} - \frac{1}{R}T(t)$$

$$\left(M_2 + \frac{I}{R^2}\right)\ddot{x} + B\dot{x} + \left(K + \frac{K_t}{R^2}\right)x = -\frac{1}{R}T(t)$$

Dinamik denge metodu:

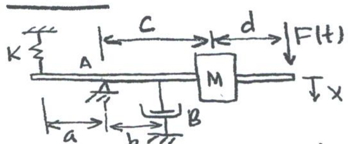


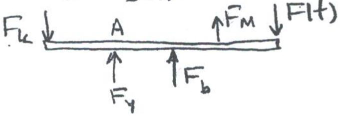
$$I\ddot{\theta} = I\left(-\frac{\ddot{x}}{R}\right)$$

$$I\frac{\ddot{x}}{R} - T_k - FR + T(t) = 0 \qquad -M\ddot{x} - F - F_k - F_b = 0$$

Sonuç yukarıdakiyle aynıdır.

3.1 u)



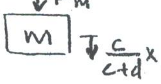


$$F_k = K\frac{a}{c+d}x$$

$$F_b = B\frac{b}{c+d}\dot{x}$$

$$\Sigma T_A = 0 = -F_k a - F_b b - F_M c + F(t)(c+d)$$

Newton kanunu:



$$M\left(\frac{c}{c+d}\ddot{x}\right) = F_M$$

$$M\left(\frac{c}{c+d}\right)\ddot{x} = -\frac{a}{c}\left(K\frac{a}{c+d}\right)x - \frac{b}{c}\left(B\frac{b}{c+d}\right)\dot{x} + \frac{c+d}{c}F(t)$$

$$M\left(\frac{c}{c+d}\right)^2\ddot{x} + B\left(\frac{b}{c+d}\right)^2\dot{x} + K\left(\frac{a}{c+d}\right)^2 x = F(t)$$

Dinamik denge metodu:

Kütlenin serbest cisim diyagramına $-x$ yönünde $\frac{c}{c+d}\ddot{x}$ kadar bir D'Alambert kuvveti eklenip statik kuvvet dengesi denklemi yazılırsa aynı sonuç bulunur.

Enerji metodu:

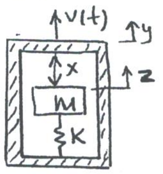
$$E_T = \frac{1}{2}K\left(\frac{a}{c+d}x\right)^2 + \frac{1}{2}M\left(\frac{c}{c+d}\dot{x}\right)^2$$

$$P = F(t)\,\dot{x} - B\left(\frac{b}{c+d}\dot{x}\right)^2 \qquad \frac{dE_T}{dt} = P$$

$$K\left(\frac{a}{c+d}\right)^2 x\dot{x} + M\left(\frac{c}{c+d}\right)^2 \dot{x}\ddot{x} = F(t)\,\dot{x} - B\left(\frac{b}{c+d}\right)^2 \dot{x}^2$$

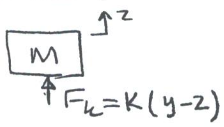
$$M\left(\frac{c}{c+d}\right)^2\ddot{x} + B\left(\frac{b}{c+d}\right)^2\dot{x} + K\left(\frac{a}{c+d}\right)^2 x = F(t)$$

3.1 v)



$\dot{y} = v(t)$

Newton yasası:



$$M\ddot{z} = F_k = K(y-z)$$

$y=0$, $z=0$ iken $x=0$ olsun.

$$x \equiv y - z \qquad z = y - x$$

$$M(\ddot{y} - \ddot{x}) = Kx$$

$$M\ddot{x} + Kx = M\ddot{y}(t)$$

$$M\ddot{x} + Kx = M\dot{v}(t)$$

Enerji metodu:

$$E_T = \frac{1}{2}K(y-z)^2 + \frac{1}{2}M\dot{z}^2 = \frac{1}{2}Kx^2 + \frac{1}{2}M(\dot{y}-\dot{x})^2$$

Çerçeve kütlesiz kabul edilirse $v(t)$'nin uygulama noktasındaki kuvvet (hız kaynağının uyguladığı kuvvet) F_k kadardır. Yani,

$$P = v(t)\,.\,F_k = \dot{y}K(y-z) = \dot{y}Kx$$

$$\frac{dE_T}{dt} = Kx\dot{x} + M(\dot{y}-\dot{x})(\ddot{y}-\ddot{x}) = \dot{y}Kx$$

$$M\cancel{(\dot{y}-\dot{x})}(\ddot{y}-\ddot{x}) = Kx\cancel{(\dot{y}-\dot{x})}$$

$$M\ddot{x} + Kx = M\ddot{y} = M\dot{v}(t)$$

3.2

3.1'de bulunan hareket denklemlerinde geçen sönüm sabitlerini ve zorlama fonksiyonlarını sıfır alarak bulunan denklemlerden sistem doğal frekansları bulunur:

a) $\omega_n = \sqrt{\dfrac{K_1K_2\left(\frac{f}{e}\right)^2}{\bar{K}M}}$ b) $\omega_n = \sqrt{\dfrac{K_1+K_2}{M+\frac{I}{r^2}}}$

c) $\omega_n = \sqrt{\dfrac{K\left(\frac{a+b}{b}\right)^2}{M+\frac{I}{r^2}\left(\frac{a+b}{b}\right)^2}}$

d) $\omega_n = \sqrt{\dfrac{K}{M+\frac{M_1}{4}+\frac{M_2}{4}+\frac{I_1}{4R^2}+\frac{I_2}{4r^2}}}$

e) $\omega_n = \sqrt{\dfrac{K_t+KR^2}{I+Mr^2}}$ f) $\omega_n = \sqrt{\dfrac{4K}{M+\frac{I}{r^2}}}$

g) $\omega_n = \sqrt{\dfrac{K}{M_1+M_2\left(\frac{f}{e}\right)^2}}$ h) $\omega_n = \sqrt{\dfrac{K}{M+\frac{I}{r^2}}}$

i) $\omega_n = \sqrt{\dfrac{K_2}{M}}$ j) $\omega_n = \sqrt{\dfrac{K_e}{M}}$

k) $\omega_n = \sqrt{\dfrac{K_1K_2\frac{e^2}{f^2}}{\left(K_2+K_1\frac{e^2}{f^2}\right)M}}$ l) $\omega_n = \sqrt{\dfrac{4K}{M_1+M_2+\frac{I_0}{r^2}}}$

m) $\omega_n = \sqrt{\dfrac{K\frac{R_2^2}{R_1^2}}{M+\frac{I_2R_2^2}{R_1^4}+\frac{I_1}{R_1^2}}}$

n) $\omega_n = \sqrt{\dfrac{\left(\frac{e}{e+f}\right)^2K}{(M/3)}}$

o) $\omega_n = \sqrt{\dfrac{16K}{M+M_1+4M_2+\frac{I_1}{R_1^2}+\frac{4I_2}{R_2^2}}}$

p) $\omega_n = \sqrt{\dfrac{K_1+K_2}{M}}$ r, s) $\omega_n = \sqrt{\dfrac{K}{M+\frac{I}{r^2}}}$

t) $\omega_n = \sqrt{\dfrac{K+\frac{K_t}{R^2}}{M_2+\frac{I}{R^2}}}$ u) $\omega_n = \dfrac{a}{c}\sqrt{\dfrac{K}{M}}$

v) $\omega_n = \sqrt{\dfrac{K}{M}}$

BÖLÜM 4

4.1

$A = 0.017\ mm^2 = 1.7\times10^{-8}\ m^2$

$\ell = 100\ m$

$\rho = 1.7\times10^{-8}\ \Omega.m$

425 $R = \rho\dfrac{\ell}{A} = 1.7\times10^{-8}\ \dfrac{100}{1.7\times10^{-8}} = 100\ \Omega$

4.2

$\alpha = 3.9\times10^{-3}\ {}^\circ C^{-1}$ $\quad \alpha = \dfrac{1}{\rho}\dfrac{\Delta\rho}{\Delta T}$

$\rho\dfrac{\ell}{A} = R \quad \Delta\rho\dfrac{\ell}{A} = \Delta R = 10\ \Omega$ oradaki fark

$\alpha = \dfrac{1}{\rho}\ \dfrac{10\frac{A}{\ell}}{\Delta T} \qquad \Delta T = \dfrac{10A}{\ell\rho\alpha}$

$\Delta T = \dfrac{10\times1.7\times10^{-8}}{100\times1.7\times10^{-8}\times3.9\times10^{-3}} = 25.6^\circ C$

$T = 20^\circ + 25.6^\circ = 45.6^\circ$

4.3

$V = 110\ VDC \quad T = 600^\circ C \quad \rho_{20} = 8\times10^{-8}\ \Omega.m$

$\alpha = 6\times10^{-3}\ {}^\circ C^{-1} \quad A = 0.1\ mm^2 = 10^{-7}\ m^2$

$P = 1\ kW$

$\alpha = \dfrac{1}{\rho}\dfrac{\Delta\rho}{\Delta T} \qquad 6\times10^{-3} = \dfrac{1}{8\times10^{-8}}\ \dfrac{\Delta\rho}{(600-20)}$

$\Delta\rho = 27.84\times10^{-8}\ \Omega.m$

$\rho_{600} = (8+27.84)\times10^{-8} = 35.84\times10^{-8}\ \Omega m$

$P = IV = \dfrac{V^2}{R} = \dfrac{110^2}{R} = 1000\ W$

$R = 12.1\ \Omega$

$12.1\ \Omega = \rho_{600}\dfrac{\ell}{A} \qquad \ell = \dfrac{12.1\times10^{-7}}{35.84\times10^{-8}} = 3.38\ m$

4.4

page 165 $C = \dfrac{2\pi\varepsilon\ell}{\ln\left(\frac{b}{a}\right)}$ $\qquad \varepsilon = 2\times10^{-11}\ F/m$

$\ell = 50\ m$

$2a = 0.5\ mm$

$2b = 6\ mm \qquad V = 5\ V$

$C = \dfrac{2\pi\times2\times10^{-11}\times50}{\ln\left(\frac{3}{0.25}\right)} = 2.53\times10^{-9}\ F$

$E = \frac{1}{2}CV^2 = 3.16\times10^{-8}\ J$

4.5

$\varepsilon = 3.1\times10^{-11}\ F/m$

$A = 0.05\times1\ m^2 \quad d = 0.1\times10^{-3} = 10^{-4}\ m$

Düz kullanılırsa,

$C_d = \varepsilon\dfrac{A}{d} = 3.1\times10^{-11}\ \dfrac{0.05}{10^{-4}} = 1.55\times10^{-8}\ F$

Rulo yapılırsa ne kapasitans iki katına çıkarak aşağıdaki gibi olur.

$C_r = 2\times1.55\times10^{-8} = 3.1\times10^{-8}\ F$

Enerji: $\quad V = 12\ V$

$E = \frac{1}{2}CV^2$

ya da $E = \frac{1}{2}\times3.1\times10^{-8}\times(12)^2 = 2.23\times10^{-6}\ J$

4.6

$L = \dfrac{\mu\ell}{2\pi}\ln\left(\dfrac{b}{a}\right) = \dfrac{4\pi\times10^{-7}\times50}{2\pi}\ln\left(\dfrac{3}{0.25}\right)$

$L = 2.48\times10^{-5}\ H$

4.7

$\ell = 5\ km = 5000\ m \quad d = 0.5\ m \quad I = 30\ A$

$a = 10\ mm = 10^{-2}\ m \quad A = \dfrac{\pi a^2}{4} = 7.854\times10^{-5}\ m^2$

$\rho = 1.7\times10^{-8}\ \Omega.m \quad \mu = 4\pi\times10^{-7}\ H/m$

a) $R = \left(\rho\dfrac{\ell}{A}\right)\times2 = 2\times1.7\times10^{-8}\times\dfrac{5000}{7.854\times10^{-5}} = 2.16\ \Omega$

$R = 2.164\ \Omega \quad P_d = I^2R = 1948\ W$

b) $L = \dfrac{\mu L}{\pi}\ln\left(\dfrac{d}{a}\right) = \dfrac{4\pi\times10^{-7}\times5000}{\pi}\ln\left(\dfrac{0.5}{0.01}\right)$

$L = 7.824\times10^{-3}\ H$

$E_m = \frac{1}{2}LI^2 = 3.52\ J$

4.8

$d_{tel} = 0.1\ mm = 10^{-4}\ m \quad n = 1000 \quad A_{tel} = 7.85\times10^{-9}\ m^2$

$d_{bobin} = 10\ mm = 10^{-2}\ m \quad \ell_{bobin} = 12\ cm = 0.12\ m$

$\mu = 4\pi\times10^{-7}\ H/m \quad \rho = 1.7\times10^{-8}\ \Omega.m$

a) $\ell_{tel} = \pi d_{bobin} n = \pi\times0.01\times1000 = 31.4\ m$

$R = \rho\dfrac{\ell_{tel}}{A_{tel}} = 1.7\times10^{-8}\times\dfrac{31.4}{7.85\times10^{-9}} = 68\ \Omega$

$L = \mu\dfrac{A_{bobin}n^2}{\ell_{bobin}} = 4\pi\times10^{-7}\ \dfrac{\pi(10^{-2})^2\ (10^3)^2}{4\ \ 0.12}\ H$

$L = 8.22\times10^{-4}\ H$

b) $I = 0.05\ A \quad P_d = I^2R = 0.17\ W$

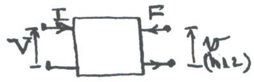
$E_m = \frac{1}{2}LI^2 = 1.027\times10^{-6}\ J$

c) $V_R = IR = 3.4\ V \qquad i(t) = 0.05\sin\omega t$

$\dfrac{di}{dt} = 0.05\omega\cos\omega t \quad V_L = 0.05\omega L = 4.11\times10^{-5}\omega$

$\dfrac{V_R}{V_L} = \dfrac{3.4}{4.11\times10^{-5}\omega} \leq 0.05 \qquad \omega \geq 1.65\times10^{6}\ rad/s$

4.10

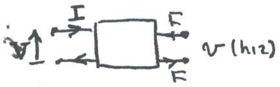


$F = CI \quad (C = \text{sabit})$

$VI + vF = 0$

$v = -\frac{1}{C}V$

4.11

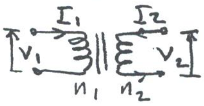


$F = CV \quad (C = \text{sabit})$

$VI + vF = 0$

$v = -\frac{1}{C}I$

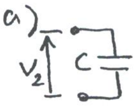
4.12



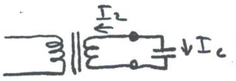
$V_2 = \frac{n_2}{n_1}V_1 \quad I_1V_1 + I_2V_2 = 0$

$I_2 = -\frac{n_1}{n_2}I_1$

a)



$I_c = C\frac{dV_2}{dt}$



$I_2 = -I_c$

$-I_2 = C\frac{dV_2}{dt} \qquad I_1\frac{n_1}{n_2} = C\frac{n_2}{n_1}\frac{dV_1}{dt}$

$I_1 = \left(\frac{n_2}{n_1}\right)^2 C\frac{dV_1}{dt}$ (Kapasitans eleman denklemi)

$I_1 = C_e\frac{dV_1}{dt}$

$C_e = \left(\frac{n_2}{n_1}\right)^2 C$

b)

$V_2 = L\frac{dI_L}{dt}$

$I_2 = -I_L$

$V_2 = -L\frac{dI_2}{dt} \qquad \frac{n_2}{n_1}V_1 = L\frac{n_1}{n_2}\frac{dI_1}{dt}$

$V_1 = \left(\frac{n_1}{n_2}\right)^2 L\frac{dI_1}{dt}$ (Endüktans eleman denklemi)

$L_e = \left(\frac{n_1}{n_2}\right)^2 L \qquad V_1 = L_e\frac{dI_1}{dt}$

c)

$V_2 = RI_R \quad I_2 = -I_R$

$V_2 = -RI_2 \qquad \frac{n_2}{n_1}V_1 = +R\frac{n_1}{n_2}I_1$

$V_1 = \left(\frac{n_1}{n_2}\right)^2 RI_1$ (Direnç eleman denklemi)

$R_e = \left(\frac{n_1}{n_2}\right)^2 R \qquad V_1 = R_eI_1$

BÖLÜM 5

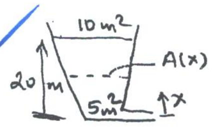
5.1

$$\frac{dE}{dt} = P_{net} = \frac{(1\times1.5 + 2\times2 - 0.5\times4 + 4\times2.5 - 3\times1)\times10^5}{3600} = \frac{10.5\times10^5}{3600}\ \text{N.m/s}$$

Bir saat sonra depolanan enerji:

$$E = P_{net}\cdot 3600 = 10.5\times10^5\ \text{N.m}$$

5.2



$A(x) = 5 + \frac{x}{20}5 \ (m^2)$

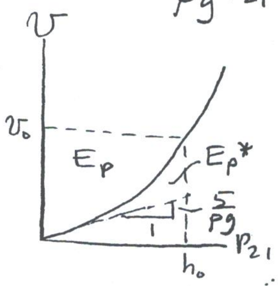
Sıvı hacmi:

$$v(h) = \int_0^h A(x)dx = \int_0^h \left(5 + \frac{1}{4}x\right)dx$$

$$v(h) = 5x + \frac{1}{4}\frac{x^2}{2}\Big|_0^h = 5h + \frac{h^2}{8}$$

$P_{21} = \rho gh \qquad h = P_{21}/\rho g$

$$v(P_{21}) = \frac{5}{\rho g}P_{21} + \frac{1}{8\rho^2g^2}P_{21}^2 \quad \text{(Nonlineer kapasitans)}$$



$$E_p^* = \int_0^{\rho gh_0} v(P_{21})dP_2$$

$$E_p^* = \int_0^{\rho gh_0}\left(\frac{5}{\rho g}P_{21} + \frac{1}{8\rho^2g^2}P_{21}^2\right)dP_{21}$$

$$E_p^* = \frac{5}{\rho g}\frac{P_{21}^2}{2} + \frac{1}{8\rho^2g^2}\frac{P_{21}^3}{3}\Big|_0^{\rho gh_0}$$

$$E_p^* = \frac{5}{\rho g}\frac{\rho^2g^2h_0^2}{2} + \frac{1}{8\rho^2g^2}\frac{\rho^3g^3h_0^3}{3} = \frac{5}{2}\rho gh_0^2 + \frac{1}{24}\rho gh_0^3$$

$$E_p = v_0h_0 - E_p^* = 5h_0^2 + \frac{1}{8}h_0^3 - E_p^*$$

$$E_p = 5h_0^2 + \frac{1}{8}h_0^3 - \frac{5}{2}\rho gh_0^2 - \frac{1}{24}\rho gh_0^3$$

(Not: Bu denklemdeki katsayıların da boyutları vardır ve bu şekilde boyut analizi sağlanmaktadır.)

Değerler yerine koyulursa sonuç nümerik olarak elde edilir.

5.3

$$E_p = \frac{1}{2}\frac{\mathcal{V}^2}{C_f} = \frac{1}{2}\frac{\mathcal{V}^2}{\frac{A}{\rho g}} = \frac{1}{2}\rho g \mathcal{V}^2 \frac{1}{A}$$

$\mathcal{V}$ sabitse E_p'yi artırmak için $A \to 0$ olmalı.

5.4

Sıvı dolu basınçlı tank:

$$C_f = \frac{\mathcal{V}_o}{\beta} = \text{sabit} \qquad Q = \frac{\mathcal{V}_o}{\beta}\frac{dP_{21}}{dt} \quad (\text{lineer})$$

$$E = \frac{1}{2}\frac{\mathcal{V}_o}{\beta}P_{21}^2 \qquad \mathcal{V}_o = 0.25\ m^3$$

$P_{21} = 200 \times 10^5\ N/m^2$ $\beta = 21000 \times 10^5\ N/m^2$

$$E_p = 0.238 \times 10^5\ N\text{-}m$$

Hava dolu basınçlı tank:

$$Q = \frac{\mathcal{V}_o}{n P_{21}}\frac{dP_{21}}{dt} \qquad \text{izotermal} : n = 1$$

$$dE_p = Q P_{21}\, dt = \frac{\mathcal{V}_o}{n} dP_{21}$$

$$E_p = \frac{\mathcal{V}_o}{n} P_{21} \qquad n = 1 \quad \mathcal{V}_o = 0.25 m^3$$

$$P_{21} = 200 \times 10^5\ N/m^2$$

$$E_p = 0.25 \times 200 \times 10^5 = 5 \times 10^6\ N\text{-}m$$

(Depolanan enerjileri karşılaştırın.)

5.5

5.2'deki çözümden de yararlanarak,

$$A(x) = 10 - \frac{x}{4} \qquad \mathcal{V}(h) = \int_0 \left(10 - \frac{x}{4}\right) dx$$

$$\mathcal{V}(h) = 10h - \frac{h^2}{8} \qquad \mathcal{V}(P_{21}) = \frac{10}{\rho g}P_{21} - \frac{1}{8\rho^2 g^2}P_{21}^2$$

(nonlineer)

$$E_p^* = \int_0^{P_{21}} \mathcal{V}(P_{21})\, dP_{21} = \frac{10}{\rho g}\frac{P_{21}^2}{2} - \frac{1}{8\rho^2 g^2}\frac{P_{21}^3}{3}\Bigg|_0^{P_{21} = \rho g h_o}$$

$$E_p^* = 5\rho g h_o^2 - \frac{1}{24}\rho g h_o^3$$

$$E_p = \mathcal{V}_o h_o - E_p^* = 10h_o^2 - \frac{h_o^3}{8} - 5\rho g h_o^2 + \frac{1}{24}\rho g h_o^3$$

Değerler konularak nümerik sonuç bulunur.

5.6

$$I = \frac{\rho \ell}{A} = \frac{1000\ kg/m^3 \times 30000 m}{0.03 m^2} = 10^9\ kg/m^4$$

$$E = \frac{1}{2} I Q^2 \qquad Q = 5\ m/s \times 0.03 m^2 = 0.15\ m^3/s$$

$$E = \frac{1}{2} \times 10^9 \times 0.15^2 = 1.125 \times 10^7\ N\text{-}m$$

Sod akışlar nortansı

$$P_{21} = I \frac{dQ}{dt}$$

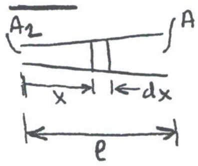
$$P_{21} = I \dot{Q}$$

$$I \frac{\Delta Q}{\Delta t} < 200 \times 10^5\ N/m^2$$

$$\Delta t > \frac{I \Delta Q}{2 \times 10^7} \qquad \Delta t > \frac{10^9 \times 0.15}{2 \times 10^7}$$

$$\Delta t > 7.5 s$$

5.7



Newton'un 2. yasası:

$$A(x)\, dp = [\rho A(x)\, dx]\frac{dV}{dt}$$

$$V = \frac{Q}{A(x)} \quad \text{ya da}$$

$$dp = \frac{\rho}{A(x)}\dot{Q}\, dx$$

$$P_2 - P_1 = \dot{Q}\rho \int_0^{\ell} \frac{dx}{A(x)} = I\dot{Q} \qquad I = \rho\int_0^{\ell} \frac{dx}{A(x)}$$

$$\text{Ama,} \quad A(x) = A_2 + (A_1 - A_2)\frac{x}{\ell}$$

$$I = \rho \int_0^{\ell} \frac{1}{A_2 + \frac{A_1 - A_2}{\ell} x} dx$$

$$= \rho\left[\frac{\ell}{A_1 - A_2} \ln\left(A_2 + \frac{A_1 - A_2}{\ell} x\right)\right]\Bigg|_{x=0}^{\ell}$$

$$I = \rho \frac{\ell}{A_1 - A_2}(\ln A_1 - \ln A_2) \quad (\text{sabit})$$

(eleman lineer)

$A_1 = 0.04\ m^2$ $A_2 = 0.01\ m^2$ $\ell = 100\ m$

$\rho = 1000\ kg/m^3$ $Q = 0.005\ m^3/s$

$$I = \rho \frac{\ell}{A_1 - A_2} \ln\left(\frac{A_1}{A_2}\right) = \frac{10^5}{0.03}\ln\left(\frac{0.04}{0.01}\right)$$

$$I = 4.62 \times 10^6\ N.s^2/m^5$$

$$E_k = \frac{1}{2} I Q^2 = 57.75\ N.m$$

5.8

$$I_1 = \frac{\rho \ell_1}{A_1} \quad I_2 = \frac{\rho \ell_2}{A_2} \quad \cdots \quad I_n = \frac{\rho \ell_n}{A_n}$$

$$\Delta P_1 = I_1 \dot{Q} \quad \Delta P_2 = I_2 \dot{Q} \quad \cdots \quad \Delta P_n = I_n \dot{Q}$$

$$P_2 - P_1 = \Delta P_1 + \Delta P_2 + \cdots + \Delta P_n =$$

$$= (I_1 + I_2 + \cdots + I_n)\dot{Q} = I_e \dot{Q}$$

$$I_e = I_1 + I_2 + \cdots + I_n$$

5.9

$$Q = C_d A_o \sqrt{\frac{2}{\rho} P_{21}} \qquad P_{21} = \frac{\rho}{2 C_d^2 A_o^2}|Q|Q$$

$$P = P_{21} Q = \frac{\rho}{2 C_d^2 A_o^2}|Q^3|$$

1 kcal = 4183 J

$P = P \cdot Q$

$P = 200 \times 10^5 \frac{N}{m^2} \times 0.001 \frac{m^3}{s} = 2 \times 10^4\ W$

Yağı ısıtmaya giden güç:

$P = (\rho Q)\rho_h \Delta T \quad \rho = 850\ kg/m^3$

$\rho_h = 0.5\ kcal/kg$

Ya da,

$2 \times 10^4 \frac{J}{s} = \underbrace{0.001 \frac{m^3}{s}}_{Q} \times \overbrace{850 \frac{kg}{m^3}}^{\rho} \times \underbrace{0.5 \frac{kcal}{kg \cdot °C}}_{} \times 4183 \frac{J}{kcal} \times \Delta T(°C)$

$\Delta T = 11.3\ °C$

5.10

$d = 25\,mm \Rightarrow 0.025\,m \quad l = 100\,m \quad \mu = 30\,cp$

$Q = 0.001\ m^3/s \quad \rho_h = 0.5 \frac{kcal}{kg\,°C}$

$\rho = 850\ kg/m^3$

$1\ poise = 1 \frac{din.s}{cm^2} \times \frac{10^{-5} N}{1\,din} \times \frac{10^4 cm^2}{1\,m^2} = 10^{-1}\ N.s/m^2$

$\mu = 30\,cp = 0.3\,p = 0.03\ N.s/m^2$

Reynolds sayısı $= \frac{4\rho Q}{\pi d \mu} = 1443 < 2000$

$R_f = \frac{128 \mu l}{\pi d^4} = 3.13 \times 10^8\ N.s/m^5$

$P_d = R_f Q^2 = 313 \frac{N.m}{s} = 313 \frac{J}{s}$

$P_d = (\rho Q)\rho_h \Delta T$

$313 \frac{J}{s} = 0.001 \frac{m^3}{s} \times 850 \frac{kg}{m^3} \times 0.5 \frac{kcal}{kg\,°C} \times 4183 \frac{J}{kcal} \Delta T(°C)$

$\Delta T = 0.176\ °C$

5.11

$Q(t) = Q_0 \sin \omega t \quad Q_0 = 0.001\ m^3/s$

$P_R = R_f Q_0 = 3.13 \times 10^8 \times 10^{-3} = 3.13 \times 10^5 \frac{N}{m^2}$

$I = \frac{\rho l}{A} = \frac{850 \times 100}{\frac{\pi}{4}(0.025)^2} = 1.73 \times 10^8 \frac{kg}{m^4}$

$P_L = I \omega Q_0 = 1.73 \times 10^8 \times 10^{-3} \omega$

$\frac{P_R}{P_L} = \frac{3.13 \times 10^5}{1.73 \times 10^5 \omega} \geq 0.05 \quad \omega < 36.2 \frac{rad}{s}$

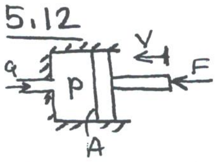
Dirence yakın davranış için: $P_R >> P_L$

$3.13 \times 10^5 >> 1.73 \times 10^5 \omega \quad \omega << 1.8 \frac{rad}{s}$

İnertansa yakın davranış için: $P_L >> P_R$

$1.73 \times 10^5 \omega >> 3.13 \times 10^5 \quad \omega >> 1.8\ rad/s$

5.12



$P = \frac{1}{A} F$

$Q = -AV$

a)

$\dot{F}_k = -KV = \dot{F}$

$A\dot{p} = -K\left(-\frac{Q}{A}\right) \quad Q = \frac{A^2}{K}\dot{p}$ (Kapasitans denklemi)

$C_e = \frac{A^2}{K}$

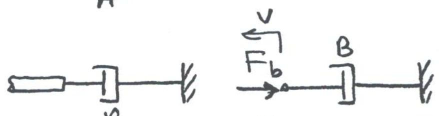
b)

$F_M = -M\dot{V} = F$

$Ap = -M\left(-\frac{\dot{Q}}{A}\right) \quad p = \frac{M}{A^2}\dot{Q}$ (İnertans denklemi)

$I_e = \frac{M}{A^2}$

c)



$F_b = -BV = F$

$Ap = -B\left(-\frac{Q}{A}\right) \quad p = \frac{B}{A^2} Q$ (Direnç denklemi)

$R_e = \frac{B}{A^2}$

5.13

a) $Q = C\omega \quad C = sabit$

$P_{12} Q + T\omega = 0$

$P_{12} C\omega + T\omega = 0$

$\left.\begin{array}{l} P_{12} = -\frac{1}{C} T \\ Q = C\omega \end{array}\right\}$ Eleman denklemleri

b) $Q = C\omega \quad C = sabit$

$P_{12} Q + T\omega = 0$

$P_{12} C\omega + T\omega = 0$

$\left.\begin{array}{l} P_{12} = -\frac{1}{C} T \\ Q = C\omega \end{array}\right\}$ Eleman denklemleri

BÖLÜM 6

6.1

$T_2 - T_1$ °K	$C_{port}\left(\frac{J}{kg \cdot K}\right)$	$\Delta E = C_p m \Delta T$ (J)
300–400	391	78200
400–600	407	162800
600–800	420	168000
800–1200	451.5	361200

$E_t = 770200$ J

$C_t = C_p m$ (nonlineer eleman)

6.2

10 kg'lık bloklar: $C_t = C_p m$ (J/K)

Al	Cu	Au	İnş T.	A.Tuğ	Cam Y.
8750	3850	1290	8350	9600	8250

0.01 m³'lük bloklar: $m = 0.01\rho$, $C_t = C_p m$

	Al	Cu	Au	İnş T.	A.Tuğ.	Cam Y.
m:	27.7	88.33	193.0	19.2	26.45	0.16
C_t:	24237	34007	24897	16032	25392	132

(m, kg; C_t, J/K cinsinden.)

6.3

$A = 1\,m^2$

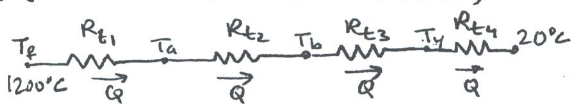
Ateş Tuğlası : $R_{t_1} = \frac{l}{KA} = \frac{0.25}{3.25\times10^{-4}T_1 + 0.795}$

Yalıtım Tuğlası : $R_{t_2} = \frac{0.15}{6.5\times10^{-5}T_2 + 0.133}$

Cam Yünü : $R_{t_3} = \frac{0.10}{(1.875\times10^{-7}T_3^2 + 9.25\times10^{-5}T_3 + 0.025)}$

Konveksiyon : $R_{t_4} = \frac{1}{Ah} = \frac{1}{\frac{3}{40}(T_y - 20) + 6}$

(R_t'ler kcal·m/h·°C cinsinden.)



T_1, T_2, T_3 sıcaklıkları katmanların ortalama sıcaklıklarıdır. Kaba bir analiz için bu sıcaklıklar her katmanın iki tarafındaki sıcaklıkların ortalaması alınırsa denklemler aşağıdaki gibi olur.

$$Q = \frac{1}{R_{t_1}}(T_f - T_a) \qquad Q = \frac{1}{R_{t_2}}(T_a - T_b)$$

$$Q = \frac{1}{R_{t_3}}(T_b - T_y) \qquad Q = \frac{1}{R_{t_4}}(T_y - 20)$$

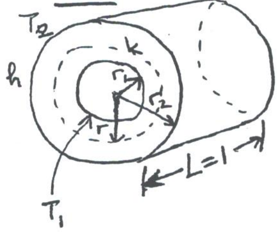
$$T_1 = \frac{T_f + T_a}{2} \qquad T_2 = \frac{T_a + T_b}{2}$$

$$T_3 = \frac{T_b + T_y}{2}$$

$$T_f = 1200°C$$

Yukarıdaki denklemler iterasyonla çözülebilir.

6.4



Isı iletimi:
Isı transferi derslerinden,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

Sınır şartları:

$r = r_1$ de $T = T_1$

$r = r_2$ de $T = T_2$

ya da $T(r) = c_1 \ln r + c_2$, ya da

$$T = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \ln\left(\frac{r}{r_2}\right) + T_2$$

$$q = -k(2\pi r L)\frac{\partial T}{\partial r} = \frac{2\pi k L}{\ln\left(\frac{r_2}{r_1}\right)}(T_1 - T_2)$$

$$= \frac{1}{R_{t_1}}(T_1 - T_2) \qquad R_{t_1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}$$

L = 1 m için

$$R_{t_1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k}$$

Konveksiyon:

$$R_h = \frac{1}{Ah} = \frac{1}{2\pi r_2 h}$$

$$R_{toplam} = R_1 + R_2 = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k} + \frac{1}{2\pi r_2 h}$$

Maksimum ısıl direnç için:

$$\frac{\partial R_{toplam}}{\partial r_2} = 0$$

$$\frac{1}{2\pi k}\frac{r_1}{r_2}\cdot\frac{1}{r_1} - \frac{1}{2\pi h r_2^2} = 0$$

$$\frac{r_2}{2\pi k} = \frac{1}{2\pi h} \qquad r_2 = \frac{k}{h}$$

6.5

Odaya giren ısıl güç: $q_g = T_s C_p \frac{Q}{\rho}$

Odadan çıkan ısıl güç: $q_c = T C_p \frac{Q}{\rho}$

$$q_{net} = \frac{C_p Q}{\rho}(T_s - T_i) = \frac{1}{R_e}(T_s - T_i)$$

(ısıl direnç) $R_e = \frac{\rho}{C_p Q}$

$$\frac{1}{R_e}(T_s - T) = C_t\frac{dT}{dt} \qquad R_e C_t \frac{dT}{dt} + T = T_s$$

$$T = T_s - (T_s - T_o)e^{-t/R_e C_t}$$

6.6

$T_i = 300°C$ $A = 1\,m^2$ $T_o = 20°$

$\rho = 8333$ kg/m³ $C_p = 340$ J/kg

$h = 16$ J/s·m²

Toplam soğuma alanı $= 2A = 2\,m^2$

$C_t = C_p M = C_p V \rho = (340 \frac{J}{kg \cdot °C})(0.01 \times 1\, m^3) \times$
$\times (8333 \frac{kg}{m^3}) = 28332 \frac{J}{°C}$

$R = \frac{1}{hA} = \frac{1}{16 \times 2} \frac{s \cdot °C}{J} = 0.03125 \frac{s \cdot C}{J}$

$C_t \frac{dT}{dt} = -\frac{1}{R}(T - T_0)$

$RC_t \frac{dT}{dt} + T = T_0 \qquad T = C_1 e^{-\frac{t}{RC_t}} + T_0$

Ama $t = 0$'da $T = 300°C$ $\quad C_1 = 300 - T_0$

$T(t) = (300 - T_0)\, e^{-t/RC_t} + T_0 \qquad T_0 = 20°C$

$T(t) = 280\, e^{-t/RC_t} + 20$

30°C olma anı:

$30 = 280\, e^{-t/RC_t} + 20$

$e^{-t/RC_t} = \frac{1}{28} \qquad e^{t/RC_t} = 28$

$\frac{t}{RC_t} = \ln 28 \qquad t = RC_t \ln 28$

$t = 2950\ s$

BÖLÜM 7

7.2

$\longrightarrow f(t) = f_0 + f_0 \sin\omega t$
$\hookrightarrow v_0 \qquad P = f_0 v_0 + f_0 v_0 \sin\omega t$

$P_{ort} = f_0 V_0$

7.3

$\longrightarrow f(t) = f_0 \sin(\omega t + \phi)$
$\hookrightarrow v(t) = v_0 \sin\omega t$

$P = f(t) v(t) = f_0 v_0 \sin(\omega t + \phi) \sin\omega t$
$= \frac{f_0 v_0}{2} [\cos\phi - \cos(2\omega t + \phi)]$

$P_{ort} = \frac{f_0 v_0}{2} \cos\phi$

7.5

$f = C \frac{dv_{21}}{dt} = f_0 \sin\omega t$

$\dot{v}_{21} = \frac{f_0}{C} \sin\omega t \qquad v_{21} = -\frac{f_0}{C\omega} \cos\omega t$

$E = \frac{1}{2} C v_{21}^2 = \frac{1}{2} \frac{f_0^2}{C\omega^2} \cos^2\omega t$

$E_{av} = \frac{1}{2} \frac{f_0^2}{C\omega^2} \left[\int_0^{2\pi/\omega} \cos^2\omega t\, dt \right] \frac{1}{\left(\frac{2\pi}{\omega}\right)}$

$E_{av} = \frac{1}{2} \frac{f_0^2}{2\pi C\omega^2} \int_0^{2\pi} \cos^2\omega t\, d(\omega t)$
$= \frac{1}{2} \frac{f_0^2}{2\pi C\omega^2} \pi = \frac{f_0^2}{4C\omega^2}$

7.6

$f = C \frac{dv_{21}}{dt} \qquad v_{21} = v_0 \sin\omega t$

$\dot{v}_{21} = v_0 \omega \cos\omega t \qquad f = C v_0 \omega \cos\omega t$

$E = \frac{1}{2} C v^2 = \frac{1}{2} C v_0^2 \sin^2\omega t$

$E_{av} = \frac{1}{2} C v_0^2 \left[\int_0^{2\pi/\omega} \sin^2\omega t\, dt \right] \frac{1}{\frac{2\pi}{\omega}}$
$= \frac{C v_0^2 \omega}{4\pi} \int_0^{2\pi/\omega} \sin^2\omega t\, dt$
$= \frac{C v_0^2}{4\pi} \int_0^{2\pi} \sin^2\omega t\, d(\omega t) = \frac{C v_0^2}{4}$

7.7

a) $v_{21} = L \frac{df}{dt} \qquad f = f_0 \sin\omega t$

$\dot{f} = f_0 \omega \cos\omega t \qquad v_{21} = L f_0 \cos\omega t$

$E = \frac{1}{2} L f^2 = \frac{1}{2} L f_0^2 \sin^2\omega t$

$E_{av} = \frac{1}{2} L f_0^2 \left[\int_0^{2\pi/\omega} \sin^2\omega t\, dt \right] \frac{1}{\frac{2\pi}{\omega}}$
$= \frac{L f_0^2 \omega}{4\pi} \int_0^{2\pi/\omega} \sin^2\omega t\, dt$
$= \frac{L f_0^2}{4\pi} \int_0^{2\pi} \sin^2\omega t\, d(\omega t) = \frac{L f_0^2}{4}$

b) $v_{21} = L \frac{df}{dt} \qquad v_{21} = v_0 \sin\omega t$

$\dot{f} = \frac{v_0}{L} \sin\omega t \qquad f = -\frac{v_0}{L\omega} \cos\omega t$

$E = \frac{1}{2} L f^2 = \frac{1}{2} \frac{v_0^2}{L\omega^2} \cos^2\omega t$

$E_{av} = \frac{1}{2} \frac{v_0^2}{L\omega^2} \left[\int_0^{2\pi/\omega} \cos^2\omega t\, dt \right] \frac{1}{\frac{2\pi}{\omega}} =$
$= \frac{v_0^2}{4\pi L\omega} \int_0^{2\pi/\omega} \cos^2\omega t\, dt$
$= \frac{v_0^2}{4\pi L\omega^2} \int_0^{2\pi} \cos^2\omega t\, d(\omega t) = \frac{v_0^2}{4L\omega^2}$

10.7a

$$(1+\sin^2 x)\ddot{x}+(1+x^2)\dot{x}+2x^3=f(t)$$

Çalışma noktası durağan durum değil. Çalışma noktasında $f(t)=f_0$

$x=x_0$, $\dot x=\dot x_0$.

$$\phi(\ddot x,\dot x,x,f)=(1+\sin^2 x)\ddot x+(1+x^2)\dot x+2x^3-f=0$$

$$(1+\sin^2 x_0)\ddot x_0+(1+x_0^2)\dot x_0+2x_0^3-f_0=0$$

$$\ddot x_0=\frac{f_0-(1+x_0^2)\dot x_0-2x_0^3}{1+\sin^2 x_0} \qquad (1)$$

$$\frac{\partial\phi}{\partial\ddot x}\Big|_0=1+\sin^2 x_0 \quad \frac{\partial\phi}{\partial\dot x}\Big|_0=1+x_0^2 \quad \frac{\partial\phi}{\partial f}\Big|_0=-1$$

$$\frac{\partial\phi}{\partial x}\Big|_0=(2\sin x_0\cos x_0)\ddot x_0+2x_0\dot x_0+6x_0^2$$

$$(1+\sin^2 x_0)\,\delta\left(\frac{d^2x}{dt^2}\right)+(1+x_0^2)\,\delta\left(\frac{dx}{dt}\right)$$
$$+(2\ddot x_0\sin x_0\cos x_0+2x_0\dot x_0+6x_0^2)\delta x-\delta f=0$$

Ama $\delta\left(\frac{d^2x}{dt^2}\right)=\frac{d^2}{dt^2}(\delta x)-\ddot x_0$

$\delta\left(\frac{dx}{dt}\right)=\frac{d}{dt}(\delta x)-\dot x_0$

Ya da,

$$(1+\sin^2 x_0)\frac{d^2}{dt^2}\delta x-(1+\sin^2 x_0)\ddot x_0$$
$$+(1+x_0^2)\frac{d}{dt}\delta x-(1+x_0^2)\dot x_0+$$
$$(2\ddot x_0\sin x_0\cos x_0+2x_0\dot x_0+6x_0^2)\delta x-\delta f=0$$

(Deviasyon denklemi)

$\frac{d^2}{dt^2}\delta x=\ddot x$ $\frac{d}{dt}\delta x=\dot x$ $\delta x=x-x_0$ $\delta f=f-f_0$

$$(1+\sin^2 x_0)\frac{d^2x}{dt^2}-(1+\sin^2 x_0)\ddot x_0+(1+x_0^2)\frac{dx}{dt}+$$
$$-(1+x_0^2)\dot x_0+(2\ddot x_0\sin x_0\cos x_0+2x_0\dot x_0+6x_0^2)x$$
$$-2x_0\ddot x_0\sin x_0\cos x_0-2x_0^2\dot x_0-6x_0^3-(f-f_0)=0$$

(Lineerleştirilmiş denklem)

Not: $\ddot x_0$ terimi (1)'deki gibi.

10.7b

$$\ddot x+(1-x^2)\dot x+x^2\sin x+x^3=f(t)$$

Çalışma noktası durağan değil.
Çalışma noktasında $f(t)=f_0$, $x=x_0$ $\dot x=\dot x_0$

$$\phi(\ddot x,\dot x,x,f)=\ddot x+(1-x^2)\dot x+x^2\sin x+x^3-f=0$$

$$\ddot x_0+(1-x_0^2)\dot x_0+x_0^2\sin x_0+x_0^3-f_0=0$$

$$\ddot x_0=f_0-(1-x_0^2)\dot x_0-x_0^2\sin x_0-x_0^3 \qquad (1)$$

$$\frac{\partial\phi}{\partial\ddot x}\Big|_0=1 \quad \frac{\partial\phi}{\partial\dot x}=(1-x_0^2) \quad \frac{\partial\phi}{\partial f}=-1$$

$$\frac{\partial\phi}{\partial x}\Big|_0=-2x_0\dot x_0+2x_0\sin x_0+x_0^2\cos x_0+3x_0^2$$

$$\delta\left(\frac{d^2x}{dt^2}\right)+(1-x_0^2)\,\delta\left(\frac{dx}{dt}\right)+$$
$$+(-2x_0\dot x_0+2x_0\sin x_0+x_0^2\cos x_0+3x_0^2)\delta x-\delta f=0$$

Ama $\delta\left(\frac{d^2x}{dt^2}\right)=\frac{d^2}{dt^2}(\delta x)-\ddot x_0$

$\delta\left(\frac{dx}{dt}\right)=\frac{d}{dt}(\delta x)-\dot x_0$

$$\frac{d^2}{dt^2}(\delta x)-\ddot x_0+(1-x_0^2)\frac{d}{dt}(\delta x)-(1-x_0^2)\dot x_0$$
$$+(-2x_0\dot x_0+2x_0\sin x_0+x_0^2\cos x_0+3x_0^2)\delta x-\delta f=0$$

(Deviasyon denklemi)

$\frac{d^2}{dt^2}\delta x=\ddot x$ $\frac{d}{dt}\delta x=\dot x$ $\delta x=x-x_0$ $\delta f=f-f_0$

$$\frac{d^2x}{dt^2}+(1-x_0^2)\frac{dx}{dt}+(-2x_0\dot x_0+2x_0\sin x_0+x_0^2\cos x_0$$
$$+3x_0^2)x-f+f_0-\ddot x_0-(1-x_0^2)\dot x_0+2x_0^2\dot x_0$$
$$-2x_0^2\sin x_0-x_0^3\cos x_0-3x_0^3=0$$

(Lineerleştirilmiş denklem)

$\ddot x_0$ terimi (1)'deki gibidir.

BÖLÜM 11

11.1

$$\mathcal{L}[e^{-at}]=\int_0^\infty e^{-st}e^{-at}dt=\int_0^\infty e^{-(s+a)t}dt$$
$$=-\frac{1}{s+a}e^{-(s+a)t}\Big|_{t=0}^{\infty}=\frac{1}{s+a}$$

$$\mathcal{L}[\sin\omega t]=\int_0^\infty e^{-st}\sin\omega t\,dt$$

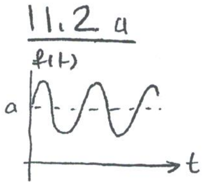
Ama $\sin\omega t=\frac{1}{2j}(e^{j\omega t}-e^{-j\omega t})$

$$\mathcal{L}[\sin\omega t]=\frac{1}{2j}\int_0^\infty(e^{(j\omega-s)t}-e^{-(j\omega+s)t})dt$$

$$=\frac{1}{2j}\left[\frac{1}{s-j\omega}e^{(j\omega-s)t}-\frac{1}{s+j\omega}e^{-(j\omega+s)t}\right]\Bigg|_{t=0}^{t=\infty}$$

$$=\frac{1}{2j}\left[\frac{1}{s-j\omega}-\frac{1}{s+j\omega}\right]=\frac{\omega}{s^2+\omega^2}$$

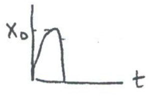
11.2 a



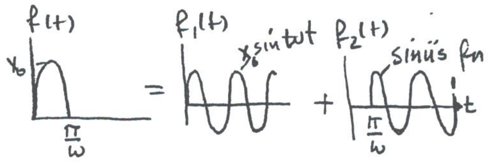
$f(t)=a+x_0\sin\omega t$ $t>0$

$$\mathcal{L}[ah(t)+x_0\sin\omega t]=$$
$$=\frac{a}{s}+\frac{x_0\omega}{s^2+\omega^2}$$

11.2 b



$f(t)=x_0\sin\omega t$ $0<t<\frac{\pi}{\omega}$



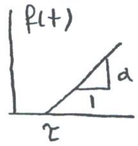
$$\mathcal{L}[f(t)] = \mathcal{L}[x_0 \sin\omega t] + \mathcal{L}[f_2 t]$$

$f_2(t)$ fonksiyonu $x_0 \sin\omega t$'nin zaman ekseni boyunca $\frac{\pi}{\omega}$ kadar kaymış hali olduğundan,

$$\mathcal{L}[f_2(t)] = e^{-\frac{\pi}{\omega}s}\,\mathcal{L}[x_0 \sin\omega t]$$

$$\mathcal{L}[f(t)] = \left(1+e^{-\frac{\pi}{\omega}s}\right)\frac{\omega x_0}{s^2+\omega^2}$$

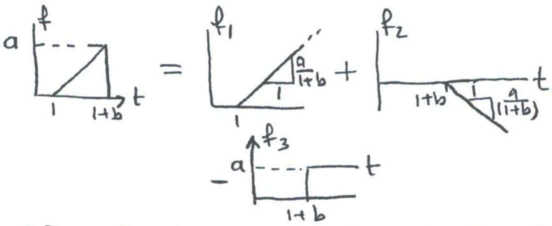
11.2c



Fonksiyon α eğimli rampa fonksiyon $\alpha r(t)$'nin τ kadar zaman ekseni boyunca kaydırılmış halidir.

$$\mathcal{L}[f(t)] = e^{-\tau s}\,\alpha\,\mathcal{L}[r(t)] = \frac{\alpha}{s^2}e^{-\tau s}$$

11.2d



$$\mathcal{L}[f(t)] = \mathcal{L}[f_1(t)] + \mathcal{L}[f_2(t)] - \mathcal{L}[f_3(t)]$$

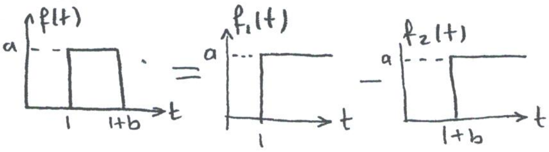
$$\mathcal{L}[f_1(t)] = e^{-s}\frac{a}{1+b}\frac{1}{s^2}$$

$$\mathcal{L}[f_2(t)] = e^{-(1+b)s}\left(-\frac{a}{1+b}\right)\frac{1}{s^2}$$

$$\mathcal{L}[f_3(t)] = e^{-(1+b)s}\frac{a}{s}$$

$$\mathcal{L}[f(t)] = \left(\frac{a}{1+b}\right)\frac{1}{s^2}\left[e^{-s} - e^{-(1+b)s}\right] - \frac{a}{s}e^{-(1+b)s}$$

11.2e

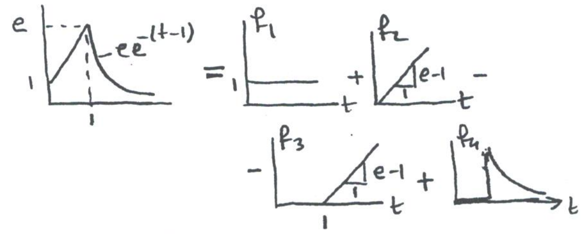


$$\mathcal{L}[f(t)] = \mathcal{L}[f_1(t)] - \mathcal{L}[f_2(t)]$$

$$= e^{-s}\frac{a}{s} - e^{-(1+b)s}\frac{a}{s}$$

$$\mathcal{L}[f(t)] = \frac{a}{s}\left[e^{-s} - e^{-(1+b)s}\right]$$

11.2f

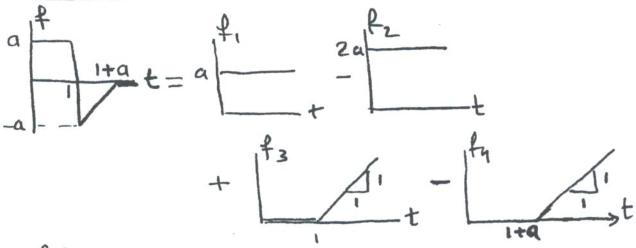


f_4 fonksiyonu ee^{-t}'nin zaman ekseni boyunca 1 kadar kaydırılmış halidir.

$$\mathcal{L}[f(t)] = \mathcal{L}[f_1] + \mathcal{L}[f_2] - \mathcal{L}[f_3] + \mathcal{L}[f_4]$$

$$= \frac{1}{s} + \frac{e-1}{s^2} - e^{-s}\frac{e-1}{s^2} + e^{-s}\frac{e}{s+1}$$

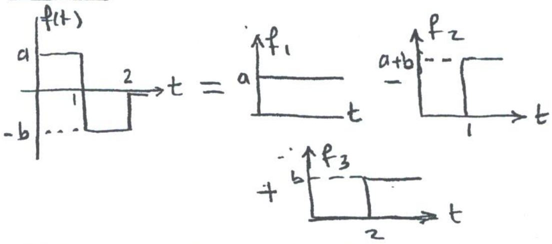
11.2g



$$\mathcal{L}[f(t)] = \mathcal{L}[f_1] - \mathcal{L}[f_2] + \mathcal{L}[f_3] + \mathcal{L}[f_4]$$

$$= \frac{a}{s} - e^{-s}\frac{2a}{s} + e^{-s}\frac{1}{s^2} - e^{-(1+a)s}\frac{1}{s^2}$$

11.2h



$$\mathcal{L}[f(t)] = \mathcal{L}[f_1] - \mathcal{L}[f_2] + \mathcal{L}[f_3]$$

$$= \frac{a}{s} - e^{-s}\frac{a+b}{s} + e^{-2s}\frac{b}{s}$$

11.3

Son değer teoreminden

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

a) $f(\infty) = \lim_{s\to 0} s\,\frac{A}{s} = A$

b) $f(\infty) = \lim_{s\to 0} s\,\frac{s+a}{(s+a)^2+\omega^2} = 0$

c) $f(\infty) = \lim_{s\to 0} s\,\frac{1}{s(s+a)^2} = \frac{1}{a^2}$

d) $f(\infty) = \lim_{s\to 0} s\,\frac{1}{s(s+a)(s+b)} = \frac{1}{ab}$

e) $f(\infty) = \lim_{s\to 0} s\,\frac{s+a}{s(s+b)(s+c)} = \frac{a}{bc}$

f) $f(\infty) = \lim_{s\to 0} s \dfrac{w_n^2}{s^2+2\xi w_n s + w_n^2} = 0$

11.4

İlk değer teoremi $f(0^+) = \lim_{s\to\infty} sF(s)$

$\mathcal{L}[\dot f(t)] = sF(s) - f(0^+)$

a) $f(0^+) = \lim_{s\to\infty} s\dfrac{1}{s+a} = 1$

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^+)$$

$$= \frac{s}{s+a} - 1 = \frac{-a}{s+a}$$

$$\dot f(0^+) = \lim_{s\to\infty} s\left(\frac{-a}{s+a}\right) = -a$$

b) $f(0^+) = \lim_{s\to\infty} s \dfrac{\tau_1 s+1}{(\tau_2 s+1)^2+w^2} = \dfrac{\tau_1}{\tau_2^2}$

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^+)$$

$$= \frac{\tau_1 s^2+s}{(\tau_2 s+1)^2+w^2} - \frac{\tau_1}{\tau_2^2} =$$

$$= \frac{\tau_1\tau_2^2 s^2 + \tau_2^2 s - \tau_1(\tau_2^2 s^2 + 2\tau_2 s+1) - \tau_1 w^2}{\tau_2^2[(\tau_2 s+1)^2+w^2]}$$

$$= \frac{\tau_2^2 s - 2\tau_1\tau_2 s - \tau_1 - \tau_1 w^2}{\tau_2^2[(\tau_2 s+1)^2+w^2]}$$

$$\dot f(0^+) = \lim_{s\to\infty} s\mathcal{L}[\dot f(t)] = \frac{\tau_2 - 2\tau_1}{\tau_2^3}$$

c) $f(0^+) = \lim_{s\to\infty} s\left[\dfrac{2s}{s^2+w^2}\right] = 2$

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^+)$$

$$= \frac{2s^2}{s^2+w^2} - 2 = \frac{-2w^2}{s^2+w^2}$$

$$\dot f(0^+) = \lim_{s\to\infty} s\mathcal{L}[\dot f(t)] = 0$$

d) $f(0^+) = \lim_{s\to\infty} s\left[\dfrac{s}{(\tau_1 s+1)(\tau_2 s+1)}\right] = \dfrac{1}{\tau_1\tau_2}$

$$\mathcal{L}[\dot f(t)] = sF(s) - f(0^+)$$

$$= \frac{s^2}{(\tau_1 s+1)(\tau_2 s+1)} - \frac{1}{\tau_1\tau_2}$$

$$= \frac{\cancel{\tau_1\tau_2 s^2} - \cancel{\tau_1\tau_2 s^2} - (\tau_1+\tau_2)s - 1}{\tau_1\tau_2(\tau_1 s+1)(\tau_2 s+1)}$$

$$\dot f(0^+) = \lim_{s\to\infty} s\mathcal{L}[\dot f(t)] = \frac{-(\tau_1+\tau_2)}{\tau_1^2\tau_2^2}$$

11.5

$f_1(t) = 1 - \cos wt \qquad F_1(s) = \dfrac{w^2}{s(s^2+w^2)}$

$$f_1(0^+) = \lim_{s\to\infty} sF_1(s) = 0$$

$$\mathcal{L}[\dot f_1(t)] = sF_1(s) - f_1(0^+) = sF_1(s)$$

$$= \frac{w^2}{s^2+w^2} = F_2(s)$$

$$f_2 = \dot f_1(t)$$

$$f_2(0^+) = \dot f_1(0^+) = \lim_{s\to\infty} s\frac{w^2}{s^2+w^2} = 0$$

$$\mathcal{L}[\dot f_2(t)] = sF_2(s) - f_2(0^+) = \frac{sw^2}{s^2+w^2}$$

$$\mathcal{L}^{-1}\left[\frac{sw^2}{s^2+w^2}\right] = \dot f_2(t) = \ddot f_1(t) = w^2\cos wt$$

11.6

$$\mathcal{L}[\underbrace{wt - \sin wt}_{f_1(t)}] = \frac{w^3}{s^2(s^2+w^2)} = F_1(s)$$

$f_1(t) = wt - \sin wt \qquad f_1(0^+) = \lim_{s\to\infty} sF_1(s) = 0$

$$\mathcal{L}[\dot f_1(t)] = sF_1(0) - f_1(0^+) = sF_1(s)$$

$$= \frac{w^3}{s(s^2+w^2)} = F_2(s)$$

$f_2(t) = \dot f_1(t) \qquad f_2(0^+) = \lim_{s\to\infty} sF_2(s) = 0$

$$\mathcal{L}[\dot f_2(t)] = sF_2(s) - f_2(0^+) = \frac{w^3}{s^2+w^2}$$

$$\mathcal{L}^{-1}\left[\frac{w^3}{s^2+w^2}\right] = \dot f_2(t) = \ddot f_1(t) = w^2\sin wt$$

11.7

$f_1(t) = t - \cos wt$

$$\mathcal{L}[f_1(t)] = \mathcal{L}[1-\cos wt] = \frac{w^2}{s(s^2+w^2)} = F_1(s)$$

$$\mathcal{L}\left[\int_0^t f_1(t)dt\right] = \frac{1}{s}F_1(s) \qquad \mathcal{L}\left[\int_0^t\int_0^t f_1(t)\,dt\,dt\right] = \frac{F_1(s)}{s^2}$$

$$\frac{w^2}{s^3(s^2+w^2)} = \frac{1}{s^2}F_1(s)$$

$$\mathcal{L}^{-1}\left[\frac{w^2}{s^3(s^2+w^2)}\right] = \mathcal{L}^{-1}\left[\frac{F_1(s)}{s^2}\right] = \int_0^t\int_0^t f_1(t)\,dt\,dt$$

$$= \int_0^t \left[t - \frac{1}{w}\sin wt\right]_{t=0}^{t=t} dt = \int_0^t \left(t - \frac{1}{w}\sin wt\right)dt$$

$$=\left[\frac{t^2}{2}+\frac{1}{\omega^2}\cos\omega t\right]_{t=0}^{t}=\frac{t^2}{2}+\frac{1}{\omega^2}\cos\omega t-\frac{1}{\omega^2}\qquad t>0$$

11.8

$$\mathcal{L}[\sin\omega t]=\frac{\omega}{s^2+\omega^2}$$

$$F(s+a)=\mathcal{L}[e^{-at}f(t)]$$

$$\mathcal{L}[e^{-at}\sin\omega t]=\frac{\omega}{(s+a)^2+\omega^2}$$

11.9a

$$\dddot{y}+6\ddot{y}+11\dot{y}+6y=2t\qquad y(0)=\dot{y}(0)=\ddot{y}(0)=0$$

iki tarafın Laplace transformu alınırsa,

$$s^3Y(s)-s^2y(0)-s\dot{y}(0)-\ddot{y}(0)$$
$$+6s^2Y(s)-6sy(0)-6\dot{y}(0)$$
$$+11sY(s)-11y(0)+6Y(s)=\frac{2}{s^2}$$

$$(s^3+6s^2+11s+6)Y(s)=\frac{2}{s^2}$$

$$Y(s)=\frac{2}{s^2(s^3+6s^2+11s+6)}$$

$$y(t)=\mathcal{L}^{-1}[Y(s)]=\mathcal{L}^{-1}\left[\frac{2}{s^2(s+1)(s+2)(s+3)}\right]$$

$$=\mathcal{L}^{-1}\left[\frac{d_1}{s}+\frac{d_2}{s^2}+\frac{d_3}{s+1}+\frac{d_4}{s+2}+\frac{d_5}{s+3}\right]$$

$$d_1=\frac{1}{1!}\frac{d}{ds}\left[s^2\frac{2}{s^2(s+1)(s+2)(s+3)}\right]\Bigg|_{s=0}=-\frac{11}{18}$$

$$d_2=s^2\frac{2}{s^2(s+1)(s+2)(s+3)}\Bigg|_{s=0}=\frac{1}{3}$$

$$d_3=(s+1)\frac{2}{s^2(s+1)(s+2)(s+3)}\Bigg|_{s=-1}=1$$

$$d_4=(s+2)Y(s)\big|_{s=-2}=-\frac{1}{2}$$

$$d_5=(s+3)Y(s)\big|_{s=-3}=\frac{1}{9}$$

$$y(t)=\mathcal{L}^{-1}\left[-\frac{11}{18s}\right]+\mathcal{L}^{-1}\left[\frac{1}{3s^2}\right]+\mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$$
$$-\mathcal{L}^{-1}\left[\frac{1}{2(s+2)}\right]+\mathcal{L}^{-1}\left[\frac{1}{9(s+3)}\right]$$

$$y(t)=-\frac{11}{18}+\frac{1}{3}t+e^{-t}-\frac{1}{2}e^{-2t}+\frac{1}{9}e^{-3t}\qquad t>0$$

11.9b

$$\dddot{y}+4\ddot{y}+4\dot{y}+3y=\overset{2h(t)\text{ (dikkat!)}}{2}\qquad y(0)=\dot{y}(0)=\ddot{y}(0)=0$$

iki tarafın Laplace transformu alınırsa,

$$s^3Y(s)-s^2y(0)-s\dot{y}(0)-\ddot{y}(0)+4s^2Y(s)-4sy(0)$$
$$-4\dot{y}(0)+4sY(s)-4y(0)+3Y(s)=\frac{2}{s}$$

$$s(s^3+4s^2+4s+3)Y(s)=2$$

$$Y(s)=\frac{2}{s(s^3+4s^2+4s+3)}=\frac{2}{(s^2+s+1)(s+3)s}$$

$$=\frac{d_1}{s+0.5+j\frac{\sqrt3}{2}}+\frac{d_2}{s+0.5-j\frac{\sqrt3}{2}}+\frac{d_3}{s+3}+\frac{d_4}{s}$$

$$d_1=\frac{2}{s(s+0.5-j\frac{\sqrt3}{2})(s+3)}\Bigg|_{s=-0.5-j\frac{\sqrt3}{2}}$$

$$=-0.286-0.33j$$

$$d_2=\frac{2}{s(s+0.5+j\frac{\sqrt3}{2})(s+3)}\Bigg|_{s=-0.5+j\frac{\sqrt3}{2}}$$

$$=-0.286+0.33j$$

$$d_3=\frac{2}{s(s^2+s+1)}\Bigg|_{s=-3}=-\frac{2}{21}\qquad d_4=\frac{2}{3}$$

$$y(t)=\mathcal{L}^{-1}\left[\frac{d_1}{s+0.5+j\frac{\sqrt3}{2}}\right]+\mathcal{L}^{-1}\left[\frac{d_2}{s+0.5-j\frac{\sqrt3}{2}}\right]$$
$$+\mathcal{L}^{-1}\left[\frac{d_3}{s+3}\right]+\mathcal{L}^{-1}\left[\frac{d_4}{s}\right]$$

$$y(t)=d_1e^{-(0.5+j\frac{\sqrt3}{2})t}+d_2e^{-(0.5-j\frac{\sqrt3}{2})t}$$
$$+d_3e^{-3t}+d_4$$

$$y(t)=d_1e^{-0.5t}\left(\cos\frac{\sqrt3}{2}t-j\sin\frac{\sqrt3}{2}t\right)$$
$$+d_2e^{-0.5t}\left(\cos\frac{\sqrt3}{2}t+j\sin\frac{\sqrt3}{2}t\right)+d_3e^{-3t}+d_4$$

$$=e^{-0.5t}\left[(d_1+d_2)\cos\frac{\sqrt3}{2}t+j(d_2-d_1)\sin\frac{\sqrt3}{2}t\right]+d_3e^{-3t}+d_4$$

$$y(t)=e^{-0.5t}\left[-0.572\cos\frac{\sqrt3}{2}t-0.66\sin\frac{\sqrt3}{2}t\right]-\frac{2}{21}e^{-3t}+0.666\qquad t>0$$

11.9c

$$\ddot{y}+\dot{y}+y=\underset{\text{Dikkat }h(t)!}{1}\qquad y(0)=-1\quad \dot{y}(0)=0$$

İki tarafın Laplace transformunu alarak,

$$s^2Y(s)-sy(0)-\dot{y}(0)+sY(s)-y(0)+Y(s)=\frac{1}{s}$$

$$(s^2+s+1)Y(s)=-s-1+\frac{1}{s}$$

$$Y(s)=\frac{-s^2-s+1}{(s^2+s+1)s}=\frac{d_1}{s+0.5+j\frac{\sqrt3}{2}}+\frac{d_2}{s+0.5-j\frac{\sqrt3}{2}}+\frac{d_3}{s}$$

$$d_1=\frac{-s^2-s+1}{s+0.5-j\frac{\sqrt3}{2}}\Bigg|_{s=-0.5-j\frac{\sqrt3}{2}}=-1-0.577j$$

$$d_2=\frac{-s^2-s+1}{s+0.5+j\frac{\sqrt3}{2}}\Bigg|_{s=-0.5+j\frac{\sqrt3}{2}}=-1+0.577j$$

$$d_3=1$$

$$y(t)=\mathcal{L}^{-1}\left[\frac{d_1}{s+0.5+j\frac{\sqrt3}{2}}\right]+\mathcal{L}^{-1}\left[\frac{d_2}{s+0.5-j\frac{\sqrt3}{2}}\right]+\mathcal{L}^{-1}\left[\frac{d_3}{s}\right]$$

$$=d_1e^{-(0.5+j\frac{\sqrt3}{2})t}+d_2e^{-(0.5-j\frac{\sqrt3}{2})t}+d_3$$

$$=d_1e^{-0.5t}\left(\cos\tfrac{\sqrt3}{2}t-j\sin\tfrac{\sqrt3}{2}t\right)$$

$$+d_2e^{-0.5t}\left(\cos\tfrac{\sqrt3}{2}t+j\sin\tfrac{\sqrt3}{2}t\right)+d_3$$

$$=e^{-0.5t}\left[(d_1+d_2)\cos\tfrac{\sqrt3}{2}t+j(d_2-d_1)\sin\tfrac{\sqrt3}{2}t\right]+d_3$$

$$y(t)=e^{-0.5t}\left(-2\cos\tfrac{\sqrt3}{2}t-1.154\sin\tfrac{\sqrt3}{2}t\right)+1 \quad (t>0)$$

11.9d

$$\ddot y+\dot y+y=\sin 2t \qquad y(0)=-1 \quad \dot y(0)=0$$

$$s^2Y(s)-sy(0)-\dot y(0)+sY(s)-y(0)+Y(s)=\frac{2}{s^2+4}$$

$$s^2Y(s)+s+sY(s)+1+Y(s)=\frac{2}{s^2+4}$$

$$(s^2+s+1)Y(s)=\frac{2}{s^2+4}-s-1=\frac{-s^3-s^2-4s-2}{s^2+4}$$

$$Y(s)=\frac{-s^3-s^2-4s-2}{(s^2+4)(s^2+s+1)}=$$

$$=\frac{d_1}{(s+2j)}+\frac{d_2}{s-2j}+\frac{d_3}{s+0.5-j\frac{\sqrt3}{2}}+\frac{d_4}{s+0.5+j\frac{\sqrt3}{2}}$$

$d_1=-0.077-0.115j$

$d_2=-0.077+0.115j$

$d_3=-0.423-0.022j$

$d_4=-0.423+0.022j$

$$y(t)=d_1\mathcal{L}^{-1}\left[\frac{1}{s+2j}\right]+d_2\mathcal{L}^{-1}\left[\frac{1}{s-2j}\right]$$

$$+d_3\mathcal{L}^{-1}\left[\frac{1}{s+0.5-j\frac{\sqrt3}{2}}\right]+d_4\mathcal{L}^{-1}\left[\frac{1}{s+0.5+j\frac{\sqrt3}{2}}\right]$$

$$=d_1e^{-j2t}+d_2e^{j2t}+d_3e^{-(0.5-j\frac{\sqrt3}{2})t}$$

$$+d_4e^{-(0.5+j\frac{\sqrt3}{2})t}=d_1(\cos 2t-j\sin 2t)$$

$$+d_2(\cos 2t+j\sin 2t)+e^{-0.5t}\left[d_3\left(\cos\tfrac{\sqrt3}{2}t\right.\right.$$

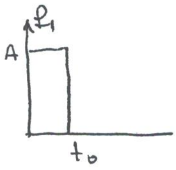
$$\left.\left.+j\sin\tfrac{\sqrt3}{2}t\right)+d_4e^{-0.5t}\left(\cos\tfrac{\sqrt3}{2}t-j\sin\tfrac{\sqrt3}{2}t\right)\right]$$

$$=(d_1+d_2)\cos 2t+j(d_2-d_1)\sin 2t$$

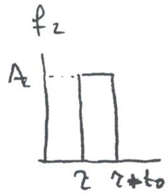
$$+e^{-0.5t}\left[(d_3+d_4)\cos\tfrac{\sqrt3}{2}t+j(d_3-d_4)\sin\tfrac{\sqrt3}{2}t\right]$$

$$y(t)=-0.154\cos 2t-0.230\sin 2t$$
$$+e^{-0.5t}\left(-0.846\cos\tfrac{\sqrt3}{2}t+0.044\sin\tfrac{\sqrt3}{2}t\right) \quad t>0$$

11.10



$$\mathcal{L}[f_1]=\frac{A}{s}(1-e^{-t_0s})$$



f_2 fonksiyonu f_1'i τ kadar zaman ekseni boyunca kaydırarak elde edilir.

$$\mathcal{L}[f_2]=e^{-\tau s}\mathcal{L}[f_1]=\frac{A}{s}(e^{-\tau s}-e^{-(t_0+\tau)s})$$

11.11a

$$F(s)=\frac{\omega^3}{s^2(s^2+\omega^2)}=\frac{\omega}{s^2}-\frac{\omega}{s^2+\omega^2}$$

$$f(t)=\mathcal{L}^{-1}\left[\frac{\omega}{s^2}-\frac{\omega}{s^2+\omega^2}\right]$$

$$f(t)=\omega t-\sin\omega t \qquad t>0$$

11.11b

$$F(s)=\frac{s^2+1}{s(s+1)^2}=\frac{d_1}{s}+\frac{d_2}{s+1}+\frac{d_3}{(s+1)^2}$$

$$d_1(s+1)^2+d_2s(s+1)+d_3s=s^2+1$$

$$d_1s^2+2d_1s+d_1+d_2s^2+d_2s+d_3s=s^2+1$$

$d_1=1 \quad d_1+d_2=1 \quad 2d_1+d_2+d_3=0$

$d_2=0 \quad d_3=-2$

$$F(s)=\frac{1}{s}-\frac{2}{(s+1)^2}$$

$$f(t)=\mathcal{L}^{-1}\left[\frac{1}{s}\right]-2\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right]$$

$$f(t)=1-2te^{-t} \qquad t>0$$

11.11c

$$F(s)=\frac{1}{s^2(s+2)}=\frac{d_1}{s}+\frac{d_2}{s^2}+\frac{d_3}{s+2}$$

$$d_1s(s+2)+d_2(s+2)+d_3s^2=1$$

$$(d_1+d_3)s^2+(2d_1+d_2)s+2d_2=1$$

$2d_2=1 \quad 2d_1+d_2=0 \quad d_2=0.5 \quad d_1=-0.25$

$d_1+d_3=0 \quad d_3=0.25$

$$f(t)=\mathcal{L}^{-1}\left[-\frac{0.25}{s}+\frac{0.5}{s^2}+\frac{0.25}{s+2}\right]$$

$$f(t)=-0.25+0.5t+0.25e^{-2t} \quad t>0$$

11.11d

$$F(s)=\frac{1}{s(s+1)(s+2)}=\frac{d_1}{s}+\frac{d_2}{s+1}+\frac{d_3}{s+2}$$

$$d_1=F(s)s\Big|_{s=0}=\frac{1}{2}$$

$$d_2=F(s)(s+1)\Big|_{s=-1}=-1$$

$$d_3=F(s)(s+2)\Big|_{s=-2}=\frac{1}{2}$$

$$F(s)=\frac{1/2}{s}-\frac{1}{s+1}+\frac{1/2}{s+2}$$

$$f(t)=\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t} \quad t>0$$

11.11e

$$F(s)=\frac{s+1}{(s+2)(s+3)}=\frac{d_1}{s+2}+\frac{d_2}{s+3}$$

$$d_1=F(s)(s+2)\Big|_{s=-2}=-1$$

$$d_2=F(s)(s+3)\Big|_{s=-3}=2$$

$$F(s)=\frac{-1}{s+2}+\frac{2}{s+3}$$

$$f(t)=\mathcal{L}^{-1}[F(s)]=-e^{-2t}+2e^{-3t} \quad t>0$$

11.11f

$$F(s)=\frac{s}{(s^2+1)(s^2+4)}=\frac{d_1s+d_2}{s^2+1}+\frac{d_3s+d_4}{s^2+4}$$

$$d_1s^3+d_2s^2+4d_1s+4d_2+d_3s^3+d_4s^2+d_3s+d_4=s$$

$d_1+d_3=0 \quad d_2+d_4=0 \quad 4d_1+d_3=1 \quad 4d_2+d_4=0$

Ya da $d_1=\frac{1}{3} \quad d_2=0 \quad d_3=-\frac{1}{3} \quad d_4=0$

$$F(s)=\frac{1}{3}\frac{s}{s^2+1}-\frac{1}{3}\frac{s}{s^2+4}$$

$$f(t)=\frac{1}{3}\cos t-\frac{1}{3}\cos 2t \quad t>0$$

11.11g

$$F(s)=\frac{1}{s(s^2+1)}=\frac{d_1}{s}+\frac{d_2s+d_3}{s^2+1}$$

$$d_1s^2+d_1+d_2s^2+d_3s=1$$

$d_1=1 \quad d_3=0 \quad d_2=-1$

$$F(s)=\frac{1}{s}-\frac{s}{s^2+1}$$

$$f(t)=1-\cos t \quad t>0$$

11.11h

$$F(s)=\frac{s^3+2s^2+3s+1}{s^2+1}=s+\frac{2s^2+2s+1}{s^2+1}$$

$$=s+2+\frac{2s-1}{s^2+1}=s+2+2\frac{s}{s^2+1}-\frac{1}{s^2+1}$$

$$f(t)=\mathcal{L}^{-1}[F(s)]=\frac{d}{dt}\delta(t)+2\delta(t)+2\cos t-\sin t \quad t>0$$

11.11i

$$F(s)=\frac{2}{s(s^2+s+1)}=\frac{d_1}{s}+\frac{d_2}{s+0.5+j\frac{\sqrt{3}}{2}}+\frac{d_3}{s+0.5-j\frac{\sqrt{3}}{2}}$$

$$d_1=F(s)s\Big|_{s=0}=2$$

$$d_2=F(s)\left(s+0.5+j\frac{\sqrt{3}}{2}\right)\Big|_{s=-0.5-j\frac{\sqrt{3}}{2}}=\frac{2}{-\frac{3}{2}+j0.5\sqrt{3}}$$

$$d_3=F(s)\left(s+0.5-j\frac{\sqrt{3}}{2}\right)\Big|_{s=-0.5+j\frac{\sqrt{3}}{2}}=\frac{2}{-\frac{3}{2}-j0.5\sqrt{3}}$$

$$d_2=\frac{2\left(-\frac{3}{2}-j0.5\sqrt{3}\right)}{3}=\frac{-3-j\sqrt{3}}{3}=-1-j\frac{1}{\sqrt{3}}$$

$$d_3=-1+j\frac{1}{\sqrt{3}}$$

$$F(s)=\frac{2}{s}-\left(1+j\frac{1}{\sqrt{3}}\right)\frac{1}{s+0.5+j\frac{\sqrt{3}}{2}}$$
$$-\left(1-j\frac{1}{\sqrt{3}}\right)\frac{1}{s+0.5-j\frac{\sqrt{3}}{2}}$$

$$f(t)=2+d_2e^{-(0.5+j\frac{\sqrt{3}}{2})t}+d_3e^{-(0.5-j\frac{\sqrt{3}}{2})t}$$

$$f(t)=2+e^{-0.5t}\left[d_2\left(\cos\frac{\sqrt{3}}{2}t-j\sin\frac{\sqrt{3}}{2}t\right)\right.$$
$$\left.+d_3\left(\cos\frac{\sqrt{3}}{2}t+j\frac{\sqrt{3}}{2}t\right)\right]$$

$$=2+e^{-0.5t}\left[(d_2+d_3)\cos\frac{\sqrt{3}}{2}t\right.$$
$$\left.+(d_3-d_2)j\sin\frac{\sqrt{3}}{2}t\right]$$

$$f(t)=2-2e^{-0.5t}\cos\frac{\sqrt{3}}{2}t+$$
$$-\frac{2}{\sqrt{3}}e^{-0.5t}\sin\frac{\sqrt{3}}{2}t$$

$$t>0$$